Name:

Math 325, Section 1

Exam 1

Show all work! How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1. (20 pts.) What must be true of a (non-empty) set $S \subseteq \mathbb{R}$ if $\inf(S) = \sup(S)$?

2. (25 pts.) Show, using the Rational Roots Theorem, that $\alpha = \sqrt{2+\sqrt{7}}$ is **not** a rational number.

3. (30 pts.) We will define a sequence $(a_n)_{n=1}^{\infty}$ by setting $a_1 = 2$, and for $n \geq 1$ (inductively) setting

$$a_{n+1} = 3 + \sqrt{2a_n}\ .$$

Show that this sequence is both monotonically increasing and bounded from above (so the sequence converges).

4. (25 pts.) Given sequences $(a_n)_{n=1}^{\infty}$ and $(b_n)_{n=1}^{\infty}$, show that <u>if</u> the sequences

$$c_n = a_n + b_n \qquad \text{and} \qquad d_n = a_n - b_n$$

both converge, <u>then</u> the sequences a_n and b_n <u>also</u> both converge!