Math 314/814, Section 5

Quiz number 8 solutions

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

Find bases for the column space and nullspace of the matrix

$$A = \begin{pmatrix} 2 & 3 & 5 & 1 \\ 2 & 1 & 1 & 2 \\ 4 & 3 & 4 & 2 \\ -2 & 1 & 3 & 0 \end{pmatrix}.$$

We, well, row reduce!

$$\begin{pmatrix} 2 & 3 & 5 & 1 \\ 2 & 1 & 1 & 2 \\ 4 & 3 & 4 & 2 \\ -2 & 1 & 3 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & 1 & 1 & 2 \\ 2 & 3 & 5 & 1 \\ 4 & 3 & 4 & 2 \\ -2 & 1 & 3 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & 1 & 1 & 2 \\ 0 & 2 & 4 & -1 \\ 4 & 3 & 4 & 2 \\ -2 & 1 & 3 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & 1 & 1 & 2 \\ 0 & 2 & 4 & -1 \\ 0 & 1 & 2 & -2 \\ -2 & 1 & 3 & 0 \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} 2 & 1 & 1 & 2 \\ 0 & 2 & 4 & -1 \\ 0 & 1 & 2 & -2 \\ 0 & 2 & 4 & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & 1 & 1 & 2 \\ 0 & 1 & 2 & -2 \\ 0 & 2 & 4 & -1 \\ 0 & 2 & 4 & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & 1 & 1 & 2 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 3 \\ 0 & 2 & 4 & 2 \end{pmatrix} \longrightarrow$$

$$\begin{pmatrix} 2 & 1 & 1 & 2 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 6 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & 1 & 1 & 2 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 6 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & 1 & 1 & 2 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & 1 & 1 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} 2 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & -1/2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

With pivots in the 1st, 2nd, and 4th columns, the corresponding columns of A,

$$\begin{pmatrix} 2 \\ 2 \\ 4 \\ -2 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 3 \\ 1 \end{pmatrix}, \text{ and } \begin{pmatrix} 1 \\ 2 \\ 2 \\ 0 \end{pmatrix},$$

are a basis for the column space of A.

For the nullspace, treating the above as a row reduction of $(A|\vec{0})$ and rehydrating, we have

$x_1 - (1/2)x_3 = 0$, $x_2 + 2x_3 = 0$, and $x_4 = 0$,

so $x_1 = (1/2)x_3$, $x_2 = -2x_3$, $x_3 = x_3$, and $x_4 = 0$.

So the nullspace of A consists of the vectors $\vec{x} = x_3 \begin{pmatrix} 1/2 \\ -2 \\ 1 \\ 0 \end{pmatrix}$, and so $\begin{pmatrix} 1/2 \\ -2 \\ 1 \\ 0 \end{pmatrix}$ is a basis for the nullspace of A.