Math 314 Matrix Theory

Exam 2 Practice Problems

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1. (20 pts.) Find, using any method (other than psychic powers), the determinant of the matrix

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 3 \\ 4 & 4 & 2 \end{pmatrix}$$

Is this matrix invertible?

2. (15 pts.) Explain why the set of vectors

$$W = \{(x, y, z) \mid x + y + 2z = 1\}$$

is **not** a subspace of $\mathbf{R}^3$.

4.(20 pts.) For the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 1 & 3 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

find bases for, and the dimensions of, the row, column, and null spaces of A.

5. (20 pts.) Find **all** of the solutions to the equation $Ax = b$, where

$$A = \begin{pmatrix} 1 & 2 & 2 & 1 \\ 2 & 4 & 3 & 3 \\ 1 & 2 & 1 & 2 \end{pmatrix} \text{ and } b = \begin{pmatrix} -2 \\ -2 \\ 0 \end{pmatrix}$$

.

5. A friend of yours runs up to you and says 'Look I've found these three vectors v_1, v_2, v_3 in $\mathbf{R}^2$ that are linearly independent!' Explain how you know, without even looking at the vectors, that your friend is wrong (again).

1. (20 pts.) Find, using any method (other than psychic powers), the determinant of the matrix

$$\begin{pmatrix} -1 & 0 & 1 & 2 \\ 2 & -1 & 1 & -1 \\ 0 & 2 & 0 & 3 \\ 1 & -1 & 1 & 1 \end{pmatrix}$$

Is this matrix invertible?

3.The system of equations

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 2 & 3 & 4 & 0 & 1 & 0 & 0 \\ 3 & 3 & -1 & -6 & 0 & 0 & 1 & 0 \\ 1 & 1 & 2 & 3 & 0 & 0 & 0 & 1 \end{array}\right) \text{row-reduces to} \left(\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 14 & -5 & -1 & 0 \\ 0 & 0 & 1 & 0 & -24 & 9 & 2 & 0 \\ 0 & 0 & 0 & 1 & 11 & -4 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right).$$

If we call the left-hand side of the first pair of matrices A, use this row-reduction information to find the dimensions and bases for the subspaces Row(A), Nul(A), and Row(A^T).

(5 pts. for each subspace.)

3. Do the vectors $\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$, and $\begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$ span $\mathbf{R^3}$?

Are they linearly independent?

Can you find a subset of this collection of vectors which forms a basis for R^3?

(10 pts. for spanning, 10 pts. for lin indep, 5 pts. for basis.)

1. (20 pts.) Use row reduction to find the determinant of the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 & -1 \\ 3 & -1 & 2 & 3 \\ 5 & 4 & 2 & 1 \\ 2 & 4 & 2 & -3 \end{pmatrix}$$

2. (20 pts.) For the vector space $\mathcal{P}_3$ of polynomials of degree less than or equal to 3, let $T : \mathcal{P}_3 \to \mathbf{R}$ be the function

$$T(p) = p(2) + p(3) .$$

Show that T is a *linear transformation*, and find numbers a, b, and c so that

$$T(x + a) = T(x^2 + b) = T(x^3 + c) = 0 .$$

4. (20 pts.) Show that the collection of vectors $W = \{(a\ b\ c)^T \in \mathbf{R}^3\ :\ 3a - 2b + c = 0\}$ is a *subspace* of $\mathbf{R}^3$, and find a *basis* for W.

$$\vec{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \text{ and } \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} .$$

5. (15 pts.) If a 5×8 matrix C has rank equal to 4, what is the dimension of its nullspace (and why does it have that value?)?

3 (25 pts) Find bases for the column , row, and nullspaces of the matrix

$$\begin{pmatrix} 1 & 2 & 1 & -1 \\ 3 & -1 & 2 & 3 \\ -3 & 8 & -1 & -9 \\ 5 & 3 & 4 & 1 \end{pmatrix}$$

1. (20 pts.) For which value(s) of x is the matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & x & 2 \\ 1 & 1 & x \end{pmatrix}$ not invertible?

2. (20 pts.) Does the collection of vectors

$$W = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \; : \; x - 2y + 4z + 3w = 0 \right\}$$

form a vector space (using the usual addition and scalar multiplication of vectors)? Explain why or why not.

3. (25 pts.) Use a superaugmented matrix to express the column space of the matrix

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 5 \\ 2 & 2 & 6 \end{pmatrix}$$

as the nullspace of another matrix B, and use this to decide if the systems of equations $A\vec{x} = \vec{b}$ are consistent, for the vectors $\vec{b}$ equal to

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 5 \\ 4 \end{pmatrix}, \text{ and } \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}.$$

4. (25 pts.) Find a collection from among the vectors

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$$

that forms a basis for $\mathbf{R}^3$, and express the remaining vectors as linear combinations of your chosen basis vectors.

[Hint: your work for the first part should tell you how to answer the second part!]

5. (10 pts.) Show why if A and B are matrices so that AB makes sense, and the matrix AB has linearly independent columns, then B must have linearly independent columns.

[Hint: What does the conclusion, about B, say about systems of linear equations?]