Math 314 Exam 1 Practice Problems

Show all work. Include all steps necessary to arrive at an answer unaided by a mechanical computational device. The steps you take to your answer are just as important, if not more important, than the answer itself. If you think it, write it!

Note: The problems below include very few resource allocation, balancing chemical equation, or network flow problems. Such problems <u>may</u> appear on our exam!

1. Use row reduction to find a solution to the following system of linear equations:

$$\begin{array}{rcrcrcr} -x & & & + & 2z & = & 3 \\ x & - & y & - & 3z & = & 3 \\ 2x & + & 3y & - & 3z & = & 6 \end{array}$$

2. Use row reduction to decide if the system of linear equations given by the augmented matrix:

$$(A|\mathbf{b}) = \left(\begin{array}{cccc|c} -1 & 0 & 2 & 3 & 2 \\ 3 & 2 & -4 & 1 & -2 \\ 0 & 3 & 0 & 1 & 6 \end{array}\right)$$

has a solution. If it does, does it have one or more than one solution?

3. Is the vector $\vec{b} = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$ in the span of the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$?

More generally, what (linear) equation among a, b, c must hold in order for $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ to be in the span of $\vec{v}_1, \vec{v}_2$?

4. Use Gauss-Jordan elimination to find the inverse of the matrix A, where

$$\text{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 3 & 5 & 8 \end{pmatrix}$$

(b) Use your answer from to find the solution to the equation $\text{A}x = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$

5. Let $\mathbf{O}$ denote the $n \times n$ matrix with all entries equal to 0.

Suppose that A and B are $n \times n$ matrices with

$$AB = \mathbf{O}\ , \qquad \text{but} \qquad B \neq \mathbf{O}\ .$$

Show that A **cannot** be invertible.
(Hint: suppose it *is*: what does that tell you about B ?)

1. Use row reduction to find a solution to the following system of linear equations:

$$\begin{array}{rcrcrcl} 4x & + & 4y & + & 2z & = & 3 \\ x & + & 3y & + & 2z & = & 1 \\ 3x & + & 2y & + & z & = & 1 \end{array}$$

2. A pet hotel can accept 50 dogs and 70 cats in its care. The average Belgian owns 2 dogs and 1 cat, while the average Luxembourgian owns 1 dog and 3 cats. How many Belgians and Luxembourgians can the pet hotel accomodate, if all of the available space is used?

3. Show that the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 3 \\ 7 \\ 4 \end{bmatrix}$, and $\vec{v}_4 = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$

are linearly dependent, and exhibit an explicit linear dependence among them.

4. Use row reduction to find the values of x for which the following matrix is **invertible**:

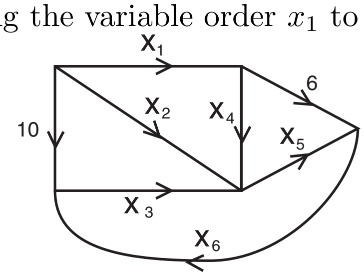
$$\text{A(x)} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & x & -1 \\ 2 & 2 & -x \end{pmatrix}$$

5. An $n \times n$ matrix A is called **nilpotent** if $A^k = 0_{n\times n}$ for some number k. Show that if A is nilpotent, then $I - A$ is an **invertible** matrix. [Hint: "factor" $I = I - A^k = (I - A)(\text{what?})$. You can try $k = 2, 3$, or 4 first to give you some feel for the general case...]

1. Use row reduction to find a solution to the following system of linear equations:

$$\begin{array}{rcrcrcl} 2x & - & y & + & 2z & = & 1 \\ x & + & y & - & 3z & = & 2 \\ 3x & - & y & + & z & = & 4 \end{array}$$

2. The figure below shows a network of pipes with rates of flow through each pipe in the indicated directions marked. Assuming no leaks, construct the augmented matrix whose solutions give the values of the unknown flow rates. [To aid your instructor, please express the (underlying) equations using the variable order x_1 to x_6 implied by the notation...]



3. For which value(s) of x are the vectors

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ x \end{bmatrix}$$

linearly **dependent**?

4. Find the inverse of the matrix

$$A = \begin{pmatrix} 3 & 1 & 2 \\ 1 & 0 & -6 \\ 7 & 2 & -3 \end{pmatrix},$$

and use this inverse to find solutions to the systems of equations $A\vec{x} = \vec{b}$, for

$$\vec{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \text{ and } \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}.$$

1. A dollar can buy

2 apples and 1 pear, or it can buy
1 apple, 2 pears and 2 grapes, or it can buy
1 apple, 1 pear, and 6 grapes.

How many pears does a dollar buy?

2. For which value(s) of x does the system of linear equations, given by the augmented matrix

$$A = \left(\begin{array}{ccc|c} 2 & 1 & 1 & 2 \\ 1 & -1 & 2 & 1 \\ 3 & 1 & x & 1 \end{array}\right),$$

have **<u>no</u>** solution?

3. Are the vectors $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 2 \\ -3 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}$ linearly independent?

4. The linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ has

$T\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $T\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

What is $T\begin{bmatrix} -1 \\ 1 \end{bmatrix}$?

(Hint: how can you express $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ in terms of $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$?)

5. Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 3 & 3 \\ 1 & 5 & 3 \end{pmatrix},$$

and use this inverse to find solutions to the systems of equations $A\vec{x} = \vec{b}$, for

$$\vec{b} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$