Math 314/814 Matrix Theory

Practice problems for Exam 2

Show all work. Include all steps necessary to arrive at an answer unaided by a mechanical computational device. The steps you take to your answer are just as important, if not more important, than the answer itself. If you think it, write it!

1. Is the collection of vectors

$$W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 : xy + z = 0 \right\}$$

a subspace of $\mathbb{R}^3$? Explain why or why not.

2. Find a basis for $\mathbb{R}^3$ that **includes** the vectors $\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$.

3. Find the **dimensions** of the row, column, and nullspaces of the matrix

$$B = \begin{bmatrix} 1 & -1 & 1 & 3 & -1 \\ 1 & 0 & 2 & 1 & 0 \\ 1 & 1 & 3 & 0 & 0 \\ 1 & 2 & 4 & 1 & -2 \end{bmatrix}$$

4. Find the determinant of the matrix $B = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 5 & 2 \\ 0 & 1 & 3 \end{bmatrix}$.

5. The matrix $A = \begin{bmatrix} -1 & 1 & 3 & -1 \\ -1 & 1 & 1 & 1 \\ -2 & 1 & 4 & -1 \\ -1 & 0 & 1 & 2 \end{bmatrix}$ has characteristic polynomial

$$\chi_A(\lambda) = (\lambda - 1)^2(\lambda - 2)^2.$$

Find the eigenvalues of A and bases for each of the corresponding eigenspaces. Is A diagonalizable? Why or why not?

6. Show that if $\vec{v} \neq \vec{0}$ is an eigenvector for two $n \times n$ matrices A and B, then $\vec{v}$ is **also** an eigenvector for both of the product matrices AB and BA. Show that under these circumstances we can conclude that the matrix $AB - BA$ is **not** invertible.