Math 314/814, Section 1

Quiz number 9 Solutions

For the matrix

$$A = \begin{pmatrix} 0 & 6 \\ 1 & 1 \end{pmatrix},$$

find a matrix P so that $P^{-1}AP = D$ is a diagonal matrix; what is D?

We start by finding the eigenvalues of A:

$$\chi_A(\lambda) = \det(A - \lambda I) = \det\begin{pmatrix} 0-\lambda & 6 \\ 1 & 1-\lambda \end{pmatrix}$$

$$= (-\lambda)(1-\lambda) - (1)(6) = \lambda^2 - \lambda - 6 = (\lambda - 3)(\lambda + 2) = 0$$

for $\lambda = 3$ and $\lambda = -2$. For each we then find an eigenbasis, by row reduction:

$$A - 3I = \begin{pmatrix} 0-3 & 6 \\ 1 & 1-3 \end{pmatrix} = \begin{pmatrix} -3 & 6 \\ 1 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & -2 \\ 1 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & -2 \\ 0 & 0 \end{pmatrix},$$

so $x - 2y = 0$, so $x = 2y$ and $\vec{v}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is a basis for the $\lambda = 3$ eigenspace.

$$A - (-2)I = \begin{pmatrix} 0+2 & 6 \\ 1 & 1+2 \end{pmatrix} = \begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix} \to \begin{pmatrix} 1 & 3 \\ 1 & 3 \end{pmatrix} \to \begin{pmatrix} 1 & 3 \\ 0 & 0 \end{pmatrix},$$

so $x + 3y = 0$, so $x = -3y$ and $\vec{v}_2 = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ is a basis for the $\lambda = -2$ eigenspace.

Consequently, $A = PDP^{-1}$ (that is, $D = P^{-1}AP$) for

$P =$ the matrix with columns equal to our eigenbases $= \begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}$,

and $D =$ the diagonal matrix having entries the corresponding eigenvalues $= \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$.

[As a check, we could multiply out PDP^{-1}, and make sure that we do get A ...]

Math 314/814, Section 5

Quiz number 9 Solutions

For the matrix

$$A = \begin{pmatrix} 0 & 6 \\ 1 & -1 \end{pmatrix} ,$$

find a matrix P so that $P^{-1}AP = D$ is a diagonal matrix; what is D?

We start by finding the eigenvalues of A:

$$\chi_A(\lambda) = \det(A - \lambda I) = \det\begin{pmatrix} 0-\lambda & 6 \\ 1 & -1-\lambda \end{pmatrix}$$

$$= (-\lambda)(-1-\lambda) - (1)(6) = \lambda^2 + \lambda - 6 = (\lambda+3)(\lambda-2) = 0$$

for $\lambda = -3$ and $\lambda = 2$. For each we then find an eigenbasis, by row reduction:

$$A - (-3)I = \begin{pmatrix} 0+3 & 6 \\ 1 & -1+3 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ 1 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix},$$

so $x + 2y = 0$, so $x = -2y$ and $\vec{v}_1 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ is a basis for the $\lambda = -3$ eigenspace.

$$A - 2I = \begin{pmatrix} 0-2 & 6 \\ 1 & -1-2 \end{pmatrix} = \begin{pmatrix} -2 & 6 \\ 1 & -3 \end{pmatrix} \to \begin{pmatrix} 1 & -3 \\ 1 & -3 \end{pmatrix} \to \begin{pmatrix} 1 & -3 \\ 0 & 0 \end{pmatrix},$$

so $x - 3y = 0$, so $x = 3y$ and $\vec{v}_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ is a basis for the $\lambda = 2$ eigenspace.

Consequently, $A = PDP^{-1}$ (that is, $D = P^{-1}AP$) for

$P =$ the matrix with columns equal to our eigenbases $= \begin{pmatrix} -2 & 3 \\ 1 & 1 \end{pmatrix}$,

and $D =$ the diagonal matrix having entries the corresponding eigenvalues $= \begin{pmatrix} -3 & 0 \\ 0 & 2 \end{pmatrix}$.

[As a check, we could multiply out PDP^{-1}, and make sure that we do get A ...]