Math 314/814, Section 1

Quiz number 7 Solutions

Find the eigenvalues of the matrix

$$A = \begin{pmatrix} 0 & 3 \\ 2 & -1 \end{pmatrix}$$

and, for each eigenvalue, find a basis for its eigenspace.

To find the eigenvalues, we need to know when

$$A - \lambda I = \begin{pmatrix} -\lambda & 3 \\ 2 & -1-\lambda \end{pmatrix}$$

has non-trivial nullspace, which will happen when

$$(-\lambda)(-1-\lambda) - (3)(2) = \lambda^2 + \lambda - 6 = (\lambda + 3)(\lambda - 2) = 0$$

so $\lambda = -3$ or $\lambda = 2$. To find bases for the eigenspaces, we find bases for the appropriate nullspaces:

$$A - (-3)I = \begin{pmatrix} 3 & 3 \\ 2 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

so we have $x + y = 0$ with y a free variable, so $x = -y$, and setting $y = 1$ we have $x = -1$ and a basis $\vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ for the (-3)-eigenspace. Similarly,

$$A - (2)I = \begin{pmatrix} -2 & 3 \\ 2 & -3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -3/2 \\ 2 & -3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -3/2 \\ 0 & 0 \end{pmatrix}$$

so we have $x - (3/2)y = 0$ with y a free variable, so $x = (3/2)y$, and setting $y = 1$ we have $x = 3/2$ and a basis $\vec{v} = \begin{pmatrix} 3/2 \\ 1 \end{pmatrix}$ for the (2)-eigenspace.

Math 314/814, Section 5

Quiz number 7 Solutions

Find the eigenvalues of the matrix

$$A = \begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix}$$

and, for each eigenvalue, find a basis for its eigenspace.

To find the eigenvalues, we need to know when

$$A - \lambda I = \begin{pmatrix} 4-\lambda & -1 \\ 2 & 1-\lambda \end{pmatrix}$$

has non-trivial nullspace, which will happen when

$$(4-\lambda)(1-\lambda) - (-1)(2) = \lambda^2 - 5\lambda + 4 + 2 = \lambda^2 - 5\lambda + 6 = (\lambda - 2)(\lambda - 3) = 0$$

so $\lambda = 2$ or $\lambda = 3$. To find bases for the eigenspaces, we find bases for the appropriate nullspaces:

$$A - 2I = \begin{pmatrix} 4-2 & -1 \\ 2 & 1-2 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & -1 \\ 2 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1/2 \\ 2 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1/2 \\ 0 & 0 \end{pmatrix} \rightarrow$$

so we have $x - (1/2)y = 0$ with y a free variable, so $x = (1/2)y$, and setting $y = 1$ we have $x = 1/2$ and a basis $\vec{v} = \begin{pmatrix} 1/2 \\ 1 \end{pmatrix}$ for the (2)-eigenspace. Similarly,

$$A - 3I = \begin{pmatrix} 4-3 & -1 \\ 2 & 1-3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \rightarrow$$

so we have $x - y = 0$ with y a free variable, so $x = y$, and setting $y = 1$ we have $x = 1$ and a basis $\vec{v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ for the (3)-eigenspace. Similarly,