Math 314/814, Section 1

Quiz number 6 Solution

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

> Research into viewer loyalty has shown that the viewing distribution of the Monday evening television programs “A”, “B”, and “C” behaves as a Markov chain, with viewers returning to the same show the following week or switching channels, according to the transition matrix
>
> $$M = \begin{pmatrix} .6 & 0 & .4 \\ .2 & .6 & .4 \\ .2 & .4 & .2 \end{pmatrix}$$

What is the steady state viewing rate for the three shows?

We want to find a vector $\vec{x}$ (with non-negative entries that sum to 1) satisfying $M\vec{x} = \vec{x}$, i.e., $(M - I)\vec{x} = \vec{0}$. So we row reduce $M - I$!

$$M - I = \begin{pmatrix} -.4 & 0 & .4 \\ .2 & -.4 & .4 \\ .2 & .4 & -.8 \end{pmatrix} \rightarrow \begin{pmatrix} -4 & 0 & 4 \\ 2 & -4 & 4 \\ 2 & 4 & -8 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & -1 \\ 2 & -4 & 4 \\ 2 & 4 & -8 \end{pmatrix} \rightarrow$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & -4 & 6 \\ 0 & 4 & -6 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & -1 \\ 0 & -4 & 6 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -3/2 \\ 0 & 0 & 0 \end{pmatrix}$$

Giving us $A - C - 0$, $B - (3/2)C - 0$, $C = C$, or $A = C$, $B = 3/2C$. $C = C$, or

$$\begin{pmatrix} A \\ B \\ C \end{pmatrix} = C \begin{pmatrix} 1 \\ 3/2 \\ 1 \end{pmatrix} .$$

The entries of this vector sum to 7/2, so to get them to sum to 1 we multiply by 2/7:

$$\begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 2/7 \\ 3/7 \\ 2/7 \end{pmatrix} \text{ is the steady state solution.}$$

Math 314/814, Section 5

Quiz number 6 Solution

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

> Research into viewer loyalty has shown that the viewing distribution of the Monday evening television programs "Q", "R", and "S" behaves as a Markov chain, with viewers returning to the same show the following week or switching channels, according to the transition matrix
>
> $$A = \begin{pmatrix} .6 & .4 & .4 \\ .2 & .6 & .4 \\ .2 & 0 & .2 \end{pmatrix}$$
>
> What is the steady state viewing rate for the three shows?

We want to find a vector $\vec{x}$ (with non-negative entries that sum to 1) satisfying $A\vec{x} = \vec{x}$, i.e., $(A - I)\vec{x} = \vec{0}$. So we row reduce $A - I$!

$$A - I = \begin{pmatrix} -.4 & .4 & .4 \\ .2 & -.4 & .4 \\ .2 & 0 & -.8 \end{pmatrix} \rightarrow \begin{pmatrix} -4 & 4 & 4 \\ 2 & -4 & 4 \\ 2 & 0 & -8 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & -1 \\ 2 & -4 & 4 \\ 2 & 0 & -8 \end{pmatrix} \rightarrow$$

$$\begin{pmatrix} 1 & -1 & -1 \\ 0 & -2 & 6 \\ 0 & 2 & -6 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & -4 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{pmatrix}$$

Giving us $Q - 4S - 0$, $R - 3S - 0$, $S = S$, or $Q = 4S$, $R = 3S$. $S = S$, or

$$\begin{pmatrix} Q \\ R \\ S \end{pmatrix} = S \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} .$$

The entries of this vector sum to 8, so to get them to sum to 1 we multiply by 1/8:

$$\begin{pmatrix} Q \\ R \\ S \end{pmatrix} = \begin{pmatrix} 1/2 \\ 3/8 \\ 1/8 \end{pmatrix} \text{ is the steady state solution.}$$