Math 314/814, Section 1

Quiz number 5 Solutions

Find bases for the column space of the matrix $A = \begin{pmatrix} 3 & 1 & -3 \\ -2 & 2 & 10 \\ 1 & 5 & 13 \end{pmatrix}$, by

(a) row reducing the matrix A :

$$A = \begin{pmatrix} 3 & 1 & -3 \\ -2 & 2 & 10 \\ 1 & 5 & 13 \end{pmatrix} \to \begin{pmatrix} 1 & 5 & 13 \\ 3 & 1 & -3 \\ -2 & 2 & 10 \end{pmatrix} \to \begin{pmatrix} 1 & 5 & 13 \\ 0 & -14 & -42 \\ 0 & 12 & 36 \end{pmatrix} \to$$

$$\begin{pmatrix} 1 & 5 & 13 \\ 0 & 1 & 3 \\ 0 & 12 & 36 \end{pmatrix} \to \begin{pmatrix} 1 & 5 & 13 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

This is in REF, and has pivots in the first two columns. So the first two columns of A,

$\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$, form a basis for col($A$).

(b) row reducing the transpose A^T of the matrix A :

$$A^T = \begin{pmatrix} 3 & -2 & 1 \\ 1 & 2 & 5 \\ -3 & 10 & 13 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 5 \\ 3 & -2 & 1 \\ -3 & 10 & 13 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 5 \\ 0 & -8 & -14 \\ 0 & 16 & 28 \end{pmatrix} \to$$

$$\begin{pmatrix} 1 & 2 & 5 \\ 0 & 1 & 7/4 \\ 0 & 16 & 28 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 5 \\ 0 & 1 & 7/4 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 3/2 \\ 0 & 1 & 7/4 \\ 0 & 0 & 0 \end{pmatrix}$$

This is in RREF, and the transposes of its non-$\vec{0}$ rows,

$\begin{pmatrix} 1 \\ 0 \\ 3/2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 7/4 \end{pmatrix}$, form a basis for $\text{row}(A^T) = \text{col}(A)$.

Math 314/814, Section 5

Quiz number 5 Solutions

Find bases for the column space of the matrix $A = \begin{pmatrix} 3 & 2 & 3 \\ -2 & 3 & 11 \\ 1 & 4 & 11 \end{pmatrix}$, by

(a) row reducing the matrix A :

$$A = \begin{pmatrix} 3 & 2 & 3 \\ -2 & 3 & 11 \\ 1 & 4 & 11 \end{pmatrix} \to \begin{pmatrix} 1 & 4 & 11 \\ 3 & 2 & 3 \\ -2 & 3 & 11 \end{pmatrix} \to \begin{pmatrix} 1 & 4 & 11 \\ 0 & -10 & -30 \\ 0 & 11 & 33 \end{pmatrix} \to$$

$$\begin{pmatrix} 1 & 4 & 11 \\ 0 & 1 & 3 \\ 0 & 11 & 33 \end{pmatrix} \to \begin{pmatrix} 1 & 4 & 11 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

This is in REF, and has pivots in the first two columns. So the first two columns of A,

$\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$, form a basis for $\mathrm{col}(A)$.

(b) row reducing the transpose A^T of the matrix A :

$$A^T = \begin{pmatrix} 3 & -2 & 1 \\ 2 & 3 & 4 \\ 3 & 11 & 11 \end{pmatrix} \to \begin{pmatrix} 2 & 3 & 4 \\ 3 & -2 & 1 \\ 3 & 11 & 11 \end{pmatrix} \to \begin{pmatrix} 1 & 3/2 & 2 \\ 3 & -2 & 1 \\ 3 & 11 & 11 \end{pmatrix} \to \begin{pmatrix} 1 & 3/2 & 2 \\ 0 & -13/2 & -5 \\ 0 & 13/2 & 5 \end{pmatrix} \to$$

$$\begin{pmatrix} 1 & 3/2 & 2 \\ 0 & 1 & 10/13 \\ 0 & 13/2 & 5 \end{pmatrix} \to \begin{pmatrix} 1 & 3/2 & 2 \\ 0 & 1 & 10/13 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 11/13 \\ 0 & 1 & 10/13 \\ 0 & 0 & 0 \end{pmatrix}$$

This is in RREF, and the transposes of its non-$\vec{0}$ rows,

$\begin{pmatrix} 1 \\ 0 \\ 11/13 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 10/13 \end{pmatrix}$, form a basis for $\mathrm{row}(A^T) = \mathrm{col}(A)$.