Math 314/814, Section 1

Quiz number 3 Solution

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

Under different conditions, ethane and atomic oxygen will combine (burn) to form differing proportions of carbon monoxide, carbon dioxide, and water.

$$C_2H_6 + O_2 \rightarrow CO + CO_2 + H_2O$$

Find at least two different (not all 0) ways to balance the equation, which create different proportions of CO versus CO_2.

Introducing variables, we write $\quad xC_2H_6 + yO_2 = zCO + uCO_2 + vH_2O$

and set the quantities of each element from each side equal.

C: $2x = z + u$, or $2x - z - u = 0$
O: $2y = z + 2u + v$, or $2y - z - 2u - v = 0$
H: $6x = 2v$, or $6x - 2v = 0$

As an augmented matrix, this is (with variable order x, y, z, u, v)

$$\left(\begin{array}{ccccc|c} 2 & 0 & -1 & -1 & 0 & 0 \\ 0 & 2 & -1 & -2 & -1 & 0 \\ 6 & 0 & 0 & 0 & -2 & 0 \end{array}\right)$$

which we row reduce:

$$\left(\begin{array}{ccccc|c} 2 & 0 & -1 & -1 & 0 & 0 \\ 0 & 2 & -1 & -2 & -1 & 0 \\ 6 & 0 & 0 & 0 & -2 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccccc|c} 2 & 0 & -1 & -1 & 0 & 0 \\ 0 & 2 & -1 & -2 & -1 & 0 \\ 0 & 0 & 3 & 3 & -2 & 0 \end{array}\right) \rightarrow$$

$$\left(\begin{array}{ccccc|c} 1 & 0 & -1/2 & -1/2 & 0 & 0 \\ 0 & 1 & -1/2 & -1 & -1/2 & 0 \\ 0 & 0 & 1 & 1 & -2/3 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -1/2 & -5/6 & 0 \\ 0 & 0 & 1 & 1 & -2/3 & 0 \end{array}\right)$$

So u and v are free, and

$x = \frac{1}{3}v$
$y = \frac{1}{2}u + \frac{5}{6}v$
$z = \frac{2}{3}v - u$

So, for example, setting (noting that using $3u \leq 2v$ gives all non-negative values, and u a multiple of 2 and v a multiple of 6 give whole number values)

$u = 0$, $v = 6$ yields $x = 2$, $y = 5$, $z = 4$, and $\quad 2C_2H_6 + 5O_2 \rightarrow 4CO + 6H_2O$

while $u = 2$, $v = 6$ yields $x = 2$, $y = 6$, $z = 2$,
and $\quad 2C_2H_6 + 6O_2 \rightarrow 2CO + 2CO_2 + 6H_2O$

Math 314/814, Section 5

Quiz number 3 Solution

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

Under different conditions, differing proportions of carbon monoxide, carbon dioxide, and atomic hydrogen will combine to form methane and water.

$$CO + CO_2 + H_2 \rightarrow CH_4 + H_2O$$

Find at least two different (not all 0) ways to balance the equation, which use different proportions of CO versus CO_2.

Introducing variables, we write $\quad xCO + yCO_2 + zH_2 = uCH_4 + vH_2O$

and set the quantities of each element from each side equal.

C: $x+y=u$, or $x+y-u=0$
O: $x+2y=v$, or $x+2y-v=0$
H: $2z=4u+2v$, or $2z-4u-2v=0$

As an augmented matrix, this is (with variable order x,y,z,u,v)

$$\begin{pmatrix} 1 & 1 & 0 & -1 & 0 & | & 0 \\ 1 & 2 & 0 & 0 & -1 & | & 0 \\ 0 & 0 & 2 & -4 & -2 & | & 0 \end{pmatrix}$$

which we row reduce:

$$\begin{pmatrix} 1 & 1 & 0 & -1 & 0 & | & 0 \\ 1 & 2 & 0 & 0 & -1 & | & 0 \\ 0 & 0 & 2 & -4 & -2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 0 & -1 & 0 & | & 0 \\ 0 & 1 & 0 & 1 & -1 & | & 0 \\ 0 & 0 & 1 & -2 & -1 & | & 0 \end{pmatrix} \rightarrow$$

$$\begin{pmatrix} 1 & 0 & 0 & -2 & 1 & | & 0 \\ 0 & 1 & 0 & 1 & -1 & | & 0 \\ 0 & 0 & 1 & -2 & -1 & | & 0 \end{pmatrix}$$

So u and v are free, and

$x = 2u - v$
$y = v - u$
$z = 2u + v$

So, for example, setting (noting that using $u \leq v \leq 2u$ gives all non-negative values)

$u=1$, $v=1$ yields $x=1$, $y=0$, $z=3$, and $\quad CO + 3H_2 \rightarrow CH_4 + H_2O$

while $u=1$, $v=2$ yields $x=0$, $y=1$, $z=4$, and $\quad CO_2 + 4H_2 \rightarrow CH_4 + 2H_2O$

More interestingly (?), $u=2$, $v=3$ yields $x=1$, $y=1$, $z=7$, and

$$CO_2 + CO + 7H_2 \rightarrow 2CH_4 + 3H_2O$$