Math 314/814, Section 1

Quiz number 1 Solution

Find a solution to the system of equations

$$\begin{aligned} 2x - y + z &= 9 \\ x - y + 3z &= 10 \\ -2x + 5y + z &= -9 \end{aligned}$$

Solution: There are any number of ways to solve this. Here is one.

Rewiting this in matrix form, and applying row reduction steps:

Start: $\begin{pmatrix} 2 & -1 & 1 & | & 9 \\ 1 & -1 & 3 & | & 10 \\ -2 & 5 & 1 & | & -9 \end{pmatrix}$

swap rows: $\begin{pmatrix} 1 & -1 & 3 & | & 10 \\ 2 & -1 & 1 & | & 9 \\ -2 & 5 & 1 & | & -9 \end{pmatrix}$

add multiples of top row: $\begin{pmatrix} 1 & -1 & 3 & | & 10 \\ 0 & 1 & -5 & | & -11 \\ -2 & 5 & 1 & | & -9 \end{pmatrix}$ $\begin{pmatrix} 1 & -1 & 3 & | & 10 \\ 0 & 1 & -5 & | & -11 \\ 0 & 3 & 7 & | & 11 \end{pmatrix}$

add multiple of middle row: $\begin{pmatrix} 1 & -1 & 3 & | & 10 \\ 0 & 1 & -5 & | & -11 \\ 0 & 0 & 22 & | & 44 \end{pmatrix}$

rescale bottom row: $\begin{pmatrix} 1 & -1 & 3 & | & 10 \\ 0 & 1 & -5 & | & -11 \\ 0 & 0 & 1 & | & 2 \end{pmatrix}$

Then we can either backsolve: $z = 2$, $y - 5z = y - 10 = -11$, so $y = -1$,
and $x - y + 3z = x - (-1) + 6 = x + 7 = 10$, so $x = 3$, or continue row reduction:

add multiples of bottom row: $\begin{pmatrix} 1 & -1 & 3 & | & 10 \\ 0 & 1 & 0 & | & -1 \\ 0 & 0 & 1 & | & 2 \end{pmatrix}$ $\begin{pmatrix} 1 & -1 & 0 & | & 4 \\ 0 & 1 & 0 & | & -1 \\ 0 & 0 & 1 & | & 2 \end{pmatrix}$

add multiple of middle row: $\begin{pmatrix} 1 & 0 & 0 & | & 3 \\ 0 & 1 & 0 & | & -1 \\ 0 & 0 & 1 & | & 2 \end{pmatrix}$

so $x = 3$, $y = -1$, $z = 2$.

So $x = 3$, $y = -1$, $z = 2$ is a solution to our original system of equations. (It is also the only solution...)

Math 314/814, Section 5

Quiz number 1 Solution

Find a solution to the system of equations

$$\begin{aligned} 3x - 2y + z &= 6 \\ 2x + y + 3z &= 11 \\ -x - y + z &= 4 \end{aligned}$$

Solution: There are any number of ways to solve this. Here is one.

Rewiting this in matrix form, and applying row reduction steps:

Start: $\begin{pmatrix} 3 & -2 & 1 & | & 6 \\ 2 & 1 & 3 & | & 11 \\ -1 & -1 & 1 & | & -4 \end{pmatrix}$

swap rows: $\begin{pmatrix} -1 & -1 & 1 & | & -4 \\ 3 & -2 & 1 & | & 6 \\ 2 & 1 & 3 & | & 11 \end{pmatrix}$ rescale top row: $\begin{pmatrix} 1 & 1 & -1 & | & 4 \\ 3 & -2 & 1 & | & 6 \\ 2 & 1 & 3 & | & 11 \end{pmatrix}$

add multiples of top row: $\begin{pmatrix} 1 & 1 & -1 & | & 4 \\ 0 & -5 & 4 & | & -6 \\ 2 & 1 & 3 & | & 11 \end{pmatrix}$ $\begin{pmatrix} 1 & 1 & -1 & | & 4 \\ 0 & -5 & 4 & | & -6 \\ 0 & -1 & 5 & | & 3 \end{pmatrix}$

swap rows, and multiply row by -1: $\begin{pmatrix} 1 & 1 & -1 & | & 4 \\ 0 & 1 & -5 & | & -3 \\ 0 & -5 & 4 & | & -6 \end{pmatrix}$

add multiple of middle row: $\begin{pmatrix} 1 & 1 & -1 & | & 4 \\ 0 & 1 & -5 & | & -3 \\ 0 & 0 & -21 & | & -21 \end{pmatrix}$

rescale bottom row: $\begin{pmatrix} 1 & 1 & -1 & | & 4 \\ 0 & 1 & -5 & | & -3 \\ 0 & 0 & 1 & | & 1 \end{pmatrix}$

Then we can either backsolve: $z = 1$, $y - 5z = y - 5 = -3$, so $y = 2$,
and $x + y - z = x + 2 - 1 = x + 1 = 4$, so $x = 3$, or continue row reduction:

add multiples of bottom row: $\begin{pmatrix} 1 & 1 & -1 & | & 4 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 1 \end{pmatrix}$ $\begin{pmatrix} 1 & 1 & 0 & | & 5 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 1 \end{pmatrix}$

add multiple of middle row: $\begin{pmatrix} 1 & 0 & 0 & | & 3 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 1 \end{pmatrix}$

so $x = 3$, $y = 2$, $z = 1$.

So $x = 3$, $y = 2$, $z = 1$ is a solution to our original system of equations. (It is also the only solution...)