Math 314/814, Section 1

Quiz number 10 Solutions

For the matrix $A = \begin{pmatrix} 3 & 5 & 8 \\ 1 & 2 & 3 \\ -2 & 3 & 1 \end{pmatrix}$, find a basis for $(\mathrm{col}(A))^\perp$, and use this to decide, for which of the vectors $\vec{b} = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 11 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 12 \end{bmatrix}$ does $A\vec{x} = \vec{b}$ have a solution?

To find the basis, we row reduce!

$$\begin{pmatrix} 3 & 5 & 8 & | & 1 & 0 & 0 \\ 1 & 2 & 3 & | & 0 & 1 & 0 \\ -2 & 3 & 1 & | & 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 & | & 0 & 1 & 0 \\ 3 & 5 & 8 & | & 1 & 0 & 0 \\ -2 & 3 & 1 & | & 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 & | & 0 & 1 & 0 \\ 0 & -1 & -1 & | & 1 & -3 & 0 \\ 0 & 7 & 7 & | & 0 & 2 & 1 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 2 & 3 & | & 0 & 1 & 0 \\ 0 & 1 & 1 & | & -1 & 3 & 0 \\ 0 & 7 & 7 & | & 0 & 2 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 & | & 0 & 1 & 0 \\ 0 & 1 & 1 & | & -1 & 3 & 0 \\ 0 & 0 & 0 & | & 7 & -19 & 1 \end{pmatrix}$$

The left-hand side is in REF, which is enough; we can read off that $(\mathrm{col}(A))^\perp = \mathrm{null}(A^T)$ has basis $\begin{bmatrix} 7 \\ -19 \\ 1 \end{bmatrix}$.

With this we can quickly check which $\vec{b}$ are in $\mathrm{col}(A)$ (so $A\vec{x} = \vec{b}$ has a solution):

$$[7 \quad -19 \quad 1] \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} = 14 - 19 + 5 = 0, \text{ so } \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} \in \mathrm{col}(A);$$

$$[7 \quad -19 \quad 1] \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} = -7 + 19 - 1 = 11 \neq 0, \text{ so } \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \notin \mathrm{col}(A);$$

$$[7 \quad -19 \quad 1] \begin{bmatrix} 1 \\ 2 \\ 11 \end{bmatrix} = 7 - 38 + 11 = -20 \neq 0, \text{ so } \begin{bmatrix} 1 \\ 2 \\ 11 \end{bmatrix} \notin \mathrm{col}(A);$$

$$[7 \quad -19 \quad 1] \begin{bmatrix} 1 \\ 1 \\ 12 \end{bmatrix} = 7 - 19 + 12 = 0 \text{ , so } \begin{bmatrix} 1 \\ 1 \\ 12 \end{bmatrix} \in \mathrm{col}(A);$$

Quiz number 10 Solutions

For the matrix $A = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \\ -3 & 5 & 8 \end{pmatrix}$, find a basis for $(\mathrm{col}(A))^\perp$, and use this to decide, for which of the vectors $\vec{b} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$ does $A\vec{x} = \vec{b}$ have a solution?

To find the basis, we row reduce!

$$\begin{pmatrix} 2 & 3 & 1 & | & 1 & 0 & 0 \\ 1 & 2 & 1 & | & 0 & 1 & 0 \\ -3 & 5 & 8 & | & 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 1 & | & 0 & 1 & 0 \\ 2 & 3 & 1 & | & 1 & 0 & 0 \\ -3 & 5 & 8 & | & 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 1 & | & 0 & 1 & 0 \\ 0 & -1 & -1 & | & 1 & -2 & 0 \\ 0 & 11 & 11 & | & 0 & 3 & 1 \end{pmatrix}$$
$$\to \begin{pmatrix} 1 & 2 & 1 & | & 0 & 1 & 0 \\ 0 & -1 & -1 & | & 1 & -2 & 0 \\ 0 & 0 & 0 & | & 11 & -19 & 1 \end{pmatrix}$$

The left-hand side is in REF, which is enough; we can read off that $(\mathrm{col}(A))^\perp = \mathrm{null}(A^T)$ has basis $\begin{bmatrix} 11 \\ -19 \\ 1 \end{bmatrix}$.

With this we can quickly check which $\vec{b}$ are in $\mathrm{col}(A)$ (so $A\vec{x} = \vec{b}$ has a solution):

$$[11 \quad -19 \quad 1] \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = 11 - 38 + 1 = -26 \neq 0, \text{ so } \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \notin \mathrm{col}(A);$$

$$[11 \quad -19 \quad 1] \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} = 22 - 19 - 3 = 0, \text{ so } \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} \in \mathrm{col}(A);$$

$$[11 \quad -19 \quad 1] \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 11 - 38 + 3 = -24 \neq 0, \text{ so } \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \notin \mathrm{col}(A);$$

$$[11 \quad -19 \quad 1] \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} = 11 - 19 + 4 = -4 \neq 0 \text{ , so } \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} \notin \mathrm{col}(A);$$