Math 314/814 Matrix Theory

Exam 2 Practice problems

Show all work. Include all steps necessary to arrive at an answer unaided by a mechanical computational device. The steps you take to your answer are just as important, if not more important, than the answer itself. If you think it, write it!

1. The company that markets Brand X toothpaste plans to introduce their product in a town where everyone uses Brand Z. Their research indicates that during each month, 2/5 of the brand X users will switch to their brand (the remainder will remain loyal), while 7/10ths of the users of their brand will switch back to Brand X. What is the transition matrix that describes this Markov process?

Based on this research, what fraction of the town's population will be Brand Z users after 3 months?

2. Find a basis for $\mathbb{R}^3$ from among the vectors

$\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}3\\2\\1\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}-1\\2\\1\end{bmatrix}$, which <u>includes</u> the vector $\begin{bmatrix}1\\1\\1\end{bmatrix}$.

3. Suppose T:$\mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation, and suppose that

$T\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}-1\\2\\3\end{bmatrix}$, $T\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}3\\-2\\1\end{bmatrix}$, and $T\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}1\\-3\\5\end{bmatrix}$.

What is $T\begin{bmatrix}2\\3\\-5\end{bmatrix}$?

4. Find the determinant of the matrix $B = \begin{bmatrix}1 & 3 & 2 & 2\\2 & 5 & 2 & 2\\0 & 2 & 3 & 4\\3 & 3 & 1 & 3\end{bmatrix}$.

5. The matrix $A = \begin{bmatrix}3 & -8 & 3\\2 & -14 & 6\\2 & -31 & 14\end{bmatrix}$ has characteristic polynomial $\chi_A(\lambda) = -(\lambda+1)(\lambda-2)^2$.

Find the eigenvalues of A and bases for each of the corresponding eigenspaces.
Is A diagonalizable? Why or why not?

6. Let V=$\mathbb{R}^3$ (3-dimensional Euclidean space) and let

$$\mathrm{W} = \{(x, y, z) \in \mathbb{R}^3 : 2x + 3y + 8z = 0\}.$$

Show that W is a **subspace** of V.

7. The system of equations

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 2 & 3 & 4 & 0 & 1 & 0 & 0 \\ 3 & 3 & -1 & -6 & 0 & 0 & 1 & 0 \\ 1 & 1 & 2 & 3 & 0 & 0 & 0 & 1 \end{array}\right) \text{row-reduces to} \left(\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 14 & -5 & -1 & 0 \\ 0 & 0 & 1 & 0 & -24 & 9 & 2 & 0 \\ 0 & 0 & 0 & 1 & 11 & -4 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right).$$

If we call the left-hand side of the first pair of matrices A, use this row-reduction information to find the dimensions and bases for the subspaces row(A), null(A), and row(A^T).

8. Do the vectors $\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$, and $\begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$ span $\mathbb{R}^3$?

Are they linearly independent?
Can you find a subset of this collection of vectors which forms a basis for $\mathbb{R}^3$?

9. Find, using any method (other than psychic powers), the determinant of the matrix

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 3 \\ 4 & 4 & 2 \end{pmatrix}$$

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Is this matrix invertible?

10. Explain why the set of vectors

$$W = \{(x, y, z) \mid x + y + 2z = 1\}$$

is **not** a subspace of $\mathbb{R}^3$.