Math 314/814, Section 6

Quiz number 8 solutions

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

Find bases for the column space and nullspace of the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 & 3 \\ 2 & 3 & 0 & 3 \\ 0 & 1 & 2 & 2 \end{pmatrix}.$$

To find the bases, we row reduce!

$$\begin{pmatrix} 1 & 2 & 1 & 3 \\ 2 & 3 & 0 & 3 \\ 0 & 1 & 2 & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & -1 & -2 & -3 \\ 0 & 1 & 2 & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & -1 \end{pmatrix} \longrightarrow$$

$$\begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The pivots are in the first, second and fourth columns, so the 1st, 2nd, and 4th columns <u>of</u> <u>A</u> are a basis for the column spaceof A:

$$\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix}$$

For the nullspace, we have one free variable, in the third column. Rehydrating the (implied) equations $A\vec{x} = \vec{0}$ from the RREF, we get

$x - 3z = 0$ and $y + 2z = 0$ and $w = 0$, so

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 3z \\ -2z \\ z \\ 0 \end{pmatrix} = z \begin{pmatrix} 3 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \text{ and so}$$

$\begin{pmatrix} 3 \\ -2 \\ 1 \\ 0 \end{pmatrix}$ is a basis for the nullspace of A.