Math 314/814, Section 6

Quiz number 7 (BUT NOT THE RIGHT PROBLEM!) Solutions

Use a superaugmented matrix to find a/the matrix B whose nullspace is equal to the column space of the matrix

$A = \begin{pmatrix} 2 & 1 & 4 \\ 1 & -2 & 7 \\ 2 & -1 & 5 \end{pmatrix}$, and use this to decide if the linear system $A\vec{x} = \vec{b}$

is consistent, for each of the vectors $\vec{b} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

OK, so your instructor made a mistake; the third column was supposed to be $3\times$first$-2\times$ second, but $3 \cdot 2 - 2 \cdot (-1) = 8$, not 5 [I can't figure out how I got 5...]. In any event, this leads to a very different answer from the one intended!

To find the matrix B, we row reduce!

$$\left(\begin{array}{ccc|ccc} 2 & 1 & 4 & 1 & 0 & 0 \\ 1 & -2 & 7 & 0 & 1 & 0 \\ 2 & -1 & 5 & 0 & 0 & 1 \end{array}\right) \Longrightarrow \left(\begin{array}{ccc|ccc} 1 & -2 & 7 & 0 & 1 & 0 \\ 2 & 1 & 4 & 1 & 0 & 0 \\ 2 & -1 & 5 & 0 & 0 & 1 \end{array}\right) \Longrightarrow$$

$$\left(\begin{array}{ccc|ccc} 1 & -2 & 7 & 0 & 1 & 0 \\ 0 & 5 & -10 & 1 & -2 & 0 \\ 0 & 3 & -9 & 0 & -2 & 1 \end{array}\right) \Longrightarrow \left(\begin{array}{ccc|ccc} 1 & -2 & 7 & 0 & 1 & 0 \\ 0 & 1 & -2 & 1/5 & -2/5 & 0 \\ 0 & 3 & -9 & 0 & -2 & 1 \end{array}\right) \Longrightarrow$$

$$\left(\begin{array}{ccc|ccc} 1 & -2 & 7 & 0 & 1 & 0 \\ 0 & 1 & -2 & 1/5 & -2/5 & 0 \\ 0 & 0 & -3 & -3/5 & -4/5 & 1 \end{array}\right) \Longrightarrow \left(\begin{array}{ccc|ccc} 1 & -2 & 7 & 0 & 1 & 0 \\ 0 & 1 & -2 & 1/5 & -2/5 & 0 \\ 0 & 0 & -15 & -3 & -4 & 5 \end{array}\right)$$

[The last step was for mostly cosmetic reasons.]

So, in row echelon form, the matrix has no row of 0's! It has a pivot in each row. This means that the original matrix A is invertible, and the columns of A span all of $\mathbf{R}^3$, and so all three of the linear systems $A\vec{x} = \vec{b}$ are consistent.

It also means that the first part of the problem, finding a matrix B with $\text{Col}(A) = \text{Null}(B)$, is somewhat meaningless; B must send every vector to the $\vec{0}$ vector, so B must be an all-zero matrix; $B = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$.