Math 314/814, Section 6

Quiz number 5 Solution

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

Use the super-augmented matrix to find the inverse of the matrix

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 1 & -2 & 2 \\ 2 & -1 & 2 \end{pmatrix}.$$

We buld the super-augmented matrix and row reduce:

$$(A|I) = \begin{pmatrix} 3 & 2 & 1 & | & 1 & 0 & 0 \\ 1 & -2 & 2 & | & 0 & 1 & 0 \\ 2 & -1 & 2 & | & 0 & 0 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & -2 & 2 & | & 0 & 1 & 0 \\ 3 & 2 & 1 & | & 1 & 0 & 0 \\ 2 & -1 & 2 & | & 0 & 0 & 1 \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} 1 & -2 & 2 & | & 0 & 1 & 0 \\ 0 & 8 & -5 & | & 1 & -3 & 0 \\ 2 & -1 & 2 & | & 0 & 0 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & -2 & 2 & | & 0 & 1 & 0 \\ 0 & 8 & -5 & | & 1 & -3 & 0 \\ 0 & 3 & -2 & | & 0 & -2 & 1 \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} 1 & -2 & 2 & | & 0 & 1 & 0 \\ 0 & 3 & -2 & | & 0 & -2 & 1 \\ 0 & 8 & -5 & | & 1 & -3 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & -2 & 2 & | & 0 & 1 & 0 \\ 0 & 1 & -2/3 & | & 0 & -2/3 & 1/3 \\ 0 & 8 & -5 & | & 1 & -3 & 0 \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} 1 & -2 & 2 & | & 0 & 1 & 0 \\ 0 & 1 & -2/3 & | & 0 & -2/3 & 1/3 \\ 0 & 0 & 1/3 & | & 1 & 7/3 & -8/3 \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} 1 & -2 & 2 & | & 0 & 1 & 0 \\ 0 & 1 & 0 & | & 2 & 4 & -5 \\ 0 & 0 & 1/3 & | & 1 & 7/3 & -8/3 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & -2 & 2 & | & 0 & 1 & 0 \\ 0 & 1 & 0 & | & 2 & 4 & -5 \\ 0 & 0 & 1 & | & 3 & 7 & -8 \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} 1 & 0 & 2 & | & 4 & 9 & -10 \\ 0 & 1 & 0 & | & 2 & 4 & -5 \\ 0 & 0 & 1 & | & 3 & 7 & -8 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 0 & | & -2 & -5 & 6 \\ 0 & 1 & 0 & | & 2 & 4 & -5 \\ 0 & 0 & 1 & | & 3 & 7 & -8 \end{pmatrix}$$

So the inverse of A is $A^{-1} = \begin{pmatrix} -2 & -5 & 6 \\ 2 & 4 & -5 \\ 3 & 7 & -8 \end{pmatrix}$.

We can check that this is correct by multiplying:

$$\begin{pmatrix} 3 & 2 & 1 \\ 1 & -2 & 2 \\ 2 & -1 & 2 \end{pmatrix} \begin{pmatrix} -2 & -5 & 6 \\ 2 & 4 & -5 \\ 3 & 7 & -8 \end{pmatrix}$$

$$= \begin{pmatrix} 3\cdot -2 + 2\cdot 2 + 1\cdot 3 & 3\cdot -5 + 2\cdot 4 + 1\cdot 7 & 3\cdot 6 + 2\cdot -5 + 1\cdot -8 \\ 1\cdot -2 - 2\cdot 2 + 2\cdot 3 & 1\cdot -5 - 2\cdot 4 + 2\cdot 7 & 1\cdot 6 - 2\cdot -5 + 2\cdot -8 \\ 2\cdot -2 - 1\cdot 2 + 2\cdot 3 & 2\cdot -5 - 1\cdot 4 + 2\cdot 7 & 2\cdot 6 - 1\cdot -5 + 2\cdot -8 \end{pmatrix}$$

$$= \begin{pmatrix} -6+4+3 & -15+8+7 & 18-10-8 \\ -2-4+6 & -5-8+14 & 6+10-16 \\ -4-2+6 & -10-4+14 & 12+5-16 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \text{ as desired.}$$