Name:

Math 314/814, Section 6

Quiz number 1 Solution

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

Use any method to find a solution to the system of equations

$$\begin{aligned} 2x + y + 2z &= 6 \\ x - y + 3z &= 8 \\ -2x - 5y + z &= 5 \end{aligned}$$

Solution: There are any number of ways to solve this. Here is one.

Rewiting this in matrix form, and applying our rules for changing the SLE without changing the solutions (i.e., row reduction):

Start: $\begin{pmatrix} 2 & 1 & 2 & | & 6 \\ 1 & -1 & 3 & | & 8 \\ -2 & -5 & 1 & | & 5 \end{pmatrix}$ Switch rows: $\begin{pmatrix} 1 & -1 & 3 & | & 8 \\ 2 & 1 & 2 & | & 6 \\ -2 & -5 & 1 & | & 5 \end{pmatrix}$

Add multiples of first row: $\begin{pmatrix} 1 & -1 & 3 & | & 8 \\ 0 & 3 & -4 & | & -10 \\ -2 & -5 & 1 & | & 5 \end{pmatrix}$ $\begin{pmatrix} 1 & -1 & 3 & | & 8 \\ 0 & 3 & -4 & | & -10 \\ 0 & -7 & 7 & | & 21 \end{pmatrix}$

Multiply 3rd rwo by $-1/7$, and switch rows:

$\begin{pmatrix} 1 & -1 & 3 & | & 8 \\ 0 & 3 & -4 & | & -10 \\ 0 & 1 & -1 & | & -3 \end{pmatrix}$ $\begin{pmatrix} 1 & -1 & 3 & | & 8 \\ 0 & 1 & -1 & | & -3 \\ 0 & 3 & -4 & | & -10 \end{pmatrix}$

Add multiples of second row:

$\begin{pmatrix} 1 & 0 & 2 & | & 5 \\ 0 & 1 & -1 & | & -3 \\ 0 & 3 & -4 & | & -10 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 & 2 & | & 5 \\ 0 & 1 & -1 & | & -3 \\ 0 & 0 & -1 & | & -1 \end{pmatrix}$

Multiply third row by -1:

$\begin{pmatrix} 1 & 0 & 2 & | & 5 \\ 0 & 1 & -1 & | & -3 \\ 0 & 0 & 1 & | & 1 \end{pmatrix}$

Add multiples of third row:

$\begin{pmatrix} 1 & 0 & 0 & | & 3 \\ 0 & 1 & -1 & | & -3 \\ 0 & 0 & 1 & | & 1 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 & 0 & | & 3 \\ 0 & 1 & 0 & | & -2 \\ 0 & 0 & 1 & | & 1 \end{pmatrix}$

which "rehydrates" to our solution: $x = 3$, $y = -2$, $z = 1$.

We can check that this *is* a solution:

$2(3) + (-2) + 2(1) = 6 - 2 + 2 = 6$
$(3) - (-2) + 3(1) = 3 + 2 + 3 = 8$
$-2(3) - 5(-2) + (1) - 6 + 10 + 1 = 5$