Math 314 Matrix Theory

Exam 2 Practice Problems

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1. (20 pts.) Find, using any method (other than psychic powers), the determinant of the matrix

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 3 \\ 4 & 4 & 2 \end{pmatrix}$$

.

Is this matrix invertible?

2. (15 pts.) Explain why the set of vectors

$$W = \{(x, y, z) \mid x + y + 2z = 1\}$$

is **not** a subspace of $\mathbf{R}^3$.

3. (25 pts.) Show that the system of equations $Ax = b$, where

$$A = \begin{pmatrix} -1 & 1 \\ 1 & -2 \\ 0 & 1 \end{pmatrix} \text{ and } b = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$$

is **not** consistent. Find the least squares solution to this sytem, i.e., the value of Ax closest to b.

4.(20 pts.) For the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 1 & 3 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

find bases for, and the dimensions of, the row, column, and null spaces of A.

5. (20 pts.) Find **all** of the solutions to the equation $Ax = b$, where

$$A = \begin{pmatrix} 1 & 2 & 2 & 1 \\ 2 & 4 & 3 & 3 \\ 1 & 2 & 1 & 2 \end{pmatrix} \text{ and } b = \begin{pmatrix} -2 \\ -2 \\ 0 \end{pmatrix}$$

.

5. A friend of yours runs up to you and says 'Look I've found these three vectors v_1, v_2, v_3 in $\mathbf{R}^2$ that are linearly independent!' Explain how you know, without even looking at the vectors, that your friend is wrong (again).

1. (20 pts.) Find, using any method (other than psychic powers), the determinant of the matrix

$$\begin{pmatrix} -1 & 0 & 1 & 2 \\ 2 & -1 & 1 & -1 \\ 0 & 2 & 0 & 3 \\ 1 & -1 & 1 & 1 \end{pmatrix}$$

.

Is this matrix invertible?

3.The system of equations

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 2 & 3 & 4 & 0 & 1 & 0 & 0 \\ 3 & 3 & -1 & -6 & 0 & 0 & 1 & 0 \\ 1 & 1 & 2 & 3 & 0 & 0 & 0 & 1 \end{array}\right) \text{row-reduces to} \left(\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 14 & -5 & -1 & 0 \\ 0 & 0 & 1 & 0 & -24 & 9 & 2 & 0 \\ 0 & 0 & 0 & 1 & 11 & -4 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right).$$

If we call the left-hand side of the first pair of matrices A, use this row-reduction information to find the dimensions and bases for the subspaces Row(A), Nul(A), and Row(A^T).

(5 pts. for each subspace.)

3. Do the vectors $\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$, and $\begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$ span $\mathbf{R}^3$?

Are they linearly independent?

Can you find a subset of this collection of vectors which forms a basis for R^3?

(10 pts. for spanning, 10 pts. for lin indep, 5 pts. for basis.)