Math 310 Homework 9
Due Tuesday, November 20

42. Find the values of $x$ which solve the system of equations

\[ x \equiv 1 \pmod{4} \]
\[ x \equiv 2 \pmod{7} \]
\[ x \equiv 3 \pmod{9} \]
\[ x \equiv 4 \pmod{13} \]

43. If $R$ and $S$ are rings show that $(r, s) \in R \times S$ is

(a) an idempotent $\iff r$ and $s$ both are

(b) nilpotent $\iff r$ and $s$ both are.

44. If $R$ is a ring, show that

\[ S = \{(r, r) : r \in R\} \subseteq R \times R \]

is a subring of $R \times R$, and $S \cong R$.

45. Show by induction that if $\varphi : R \to S$ is a homomorphism, $x \in R$, and $n \in \mathbb{N}$, then

(a) $\varphi(n \cdot x) = n \cdot \varphi(x)$

(b) $\varphi(x^n) = (\varphi(x))^n$

46. Show that $\mathbb{Z}_2 \times \mathbb{Z}_8$ and $\mathbb{Z}_4 \times \mathbb{Z}_4$ are not isomorphic. Show, however, that the two rings have the same number of units, zero divisors, idempotents, and nilpotent elements!

(Hint (for the first part): where must $4 \cdot (1, 1)$ be sent, under a homomorphism from $\mathbb{Z}_2 \times \mathbb{Z}_8$ to $\mathbb{Z}_4 \times \mathbb{Z}_4$?)