Math 310 Homework 8
Due Tuesday, November 13

38. Show that if $R \cong S$ and $R$ is an integral domain, then so is $S$.

39. If $R$ is a ring with $0_R \neq 1_R$, then an element $a \in R$ cannot be both a zero divisor and a unit.

40. Show that the ring $\mathbb{Z}_6[i] = \{a + bi : a, b \in \mathbb{Z}_6\}$, with addition and multiplication defined as in problem 36, is not a field.

41. Show that “is isomorphic to” is an equivalence relation, i.e., for any three rings $R$, $S$, and $T$,
   (a) $R \cong R$
   (b) If $R \cong S$, then $S \cong R$
   (c) If $R \cong S$ and $S \cong T$, then $R \cong T$

   (Hint: the “obvious” functions work, but don’t forget to show that each is both bijective and a homomorphism!)

For Math 310H, or extra credit:

H5. Let $n$ be a positive integer that is not the square of another integer (so that $\sqrt{n}$ is not rational). Let

$$\mathbb{Q}[\sqrt{n}] = \{a + b\sqrt{n} : a, b \in \mathbb{Q}\} \subseteq \mathbb{R}$$

with the usual addition and multiplication from $\mathbb{R}$. Show that $\mathbb{Q}[\sqrt{n}]$ is a subfield of $\mathbb{R}$, i.e., it is both a subring and a field in its own right.