Math 310 Homework 7

Due Tuesday, November 6

33. (Childs, p123, E12) Let R be a ring with identity. Show that

    (a) $(-1) \cdot (-1) = 1$

    (b) $-(-a) = a$

    (c) for every $a, b \in R$, $(-a) \cdot b = -(a \cdot b)$

34. (Childs, p.123, E13) Which of the axioms for a ring, integral domain, or field **fail** for the following sets?

    (a) the natural numbers $\mathbb{N}$

    (b) the non-negative real numbers $\mathbb{R}_+$

35. (a converse to part of Problem 29) Suppose R is a ring, and a and b are elements of R, with $a \neq 0_R$. Show that if the equation

$$ax = b$$

    has more than one solution, then a is a zero divisor!

36. (Childs, p.126, E7) Let R be the set of elements of the form $a+b\mathrm{i}$, where $a, b \in \mathbb{Z}_3$ and i is a symbol satisfying $\mathrm{i}^2 = -1$. If we add and multiply these objects, just like we do in the complex numbers $\mathbb{C}$, we get a commutative ring, which we will call $\mathbb{F}_9$. (You do **not** need to show this!) This ring has 9 elements.

    (a) Write down these 9 elements.

    (b) Show that every element of $\mathbb{F}_9$ has a multiplicative inverse, so that $\mathbb{F}_9$ is a field.

37. Show that if $f : R \to S$ is a homomorphism, then

$$f(R) = \{s \in S : s = f(r) \text{ for some } r \in R\}$$

    is a subring of S .