Math 310 Homework 6

Due Tuesday, October 30

28. (Childs, p.121, E2) Suppose R is a ring with no zero divisors, and S is a subring of R. Show that S has no zero divisors.

29. (Childs, p.121, E3) Suppose R is a ring and $a \in R$ is a zero divisor. If $b \in R$, show that the equation

$$ax = b$$

has *either* no solution *or* has more than one solution. (I.e., show that *if* it has a solution, *then* it has more than one solution.)

30. For p a prime number, let

$$\mathbb{Q}_p = \{a/b : \, a, b \in \mathbb{Z} \text{ where } p \nmid b\} \subseteq \mathbb{Q}$$

Show that $\mathbb{Q}_p$ is a subring of $\mathbb{Q}$.

31. A *Boolean ring* is a ring R where for every $x \in R$ we have $x^2 = x$. Show that if R is a Boolean ring, then

(a) $r + r = 0_R$ for every $r \in R$. (Hint: look at $(r + r)^2$.)

(b) R is commutative . (Hint: for $r, s \in R$, look at $(r + s)^2$.

32. Suppose that R is a set on which we have a notion of addition and multiplication, and we have shown that it satisfies every axiom for a ring *except* that addition is commutative. Show that then addition *must* be commutative! I.e., show that together all of the other properties of a ring *imply* that $a + b = b + a$.

(Hint: compute $(a + b)(1_R + 1_R)$, in two different ways, using the distributive law.)

For Math 310H, or extra credit:

H4. Let S be any set, and let $P(S) = \{A : \, A \subseteq S\}$ be the set of all subsets of S. Then define, for any $A, B \in P(S)$,

$A + B = (A \setminus B) \cup (B \setminus A)$ and $AB = A \cap B$.

Show that, with this addition and multiplication, $P(S)$ is a commutative ring.

Hint/instructions: verify the axioms by drawing pictures, i.e., Venn diagrams. For example, we have: