Math 310 Homework 3  
Due Tuesday, September 25

13. Use the Euclidean Algorithm to find the gcd of $a = 1111$ and $b = 473$, then reverse the steps of your calculations to write $\text{gcd}(a, b)$ as a combination of $a$ and $b$.

14. Repeat problem 13, with the numbers $a = 1357$ and $b = 2468$.

15. (Childs, p.49, E1) Show by induction that if a prime number $p$ divides the product $a_1 \cdots a_n$ of $n$ integers $a_1, \ldots, a_n$, then $p$ divides at least one of the $a_i$.

16. Show that if $a$ is an integer, $n \geq 1$, $p$ is prime, and $p|a^n$, then $p^n|a^n$.

17. (Childs, p.50, E3) Show that if $n \geq 1$ is not prime, then $n$ can be factored as $n = pq$ where $p$ is prime and $p \leq \sqrt{n}$. Use this to determine whether or not 239 is prime.

For Math 310H, or extra credit:

H2. (Childs, p.51, E5) Show that if $a$ and $b$ are integers, both $\geq 1$ and with $(a, b) = 1$, and $ab = c^r$, then $a = x^r$ and $b = y^r$ for some integers $x$ and $y$.

(Hint: Ignore Childs’ hint, he was trying to be too clever. One approach is to use complete induction (on $c$).)