Math 310 Homework 1
Due Tuesday, September 11

1. (Childs, p11, E2) Use mathematical induction to show that for every \( n \geq 1 \),
\[
\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + \cdots + n^3 = \left( \frac{n(n+1)}{2} \right)^2 .
\]

2. Use mathematical induction to show that, for every \( n \geq 0 \),
\[
4 \cdot 5^n + 7 \cdot (27)^n \text{ is a multiple of } 11. 
\]

3. Use mathematical induction to show that, for every \( n \geq 0 \),
\[
55 \cdot (44)^n - 6 \cdot (23)^n \text{ is a multiple of } 7 .
\]

4. (Childs, p12, E8) Use mathematical induction to show that for every odd number \( m \geq 1 \),
\[
4^m + 5^m \text{ is a multiple of } 9 .
\]
(Hint: don’t induct on \( m \) !)

5. (Childs, p.15, E4) Use complete induction to show that for any convex polygon with \( n \) sides, the sum of the angles inside the polygon is (in radians) \( (n - 2)\pi \). [FYI: convex means, essentially, that the line segment running between any two non-adjacent vertices of the polygon lies entirely on the inside of the polygon.]

6. (Childs, p.18, E3) Suppose that, for some fixed integer \( N \), a set \( S \) of integers has the property that \( s \leq N \) for every \( s \in S \). Show that \( S \) has a largest element, i.e., there is an \( s_0 \in S \) so that \( s \leq s_0 \) for every \( s \in S \). (Hint: read Childs’ hint!)