Math 310 Homework 1

Due Tuesday, September 11

1. (Childs, p11, E2) Use mathematical induction to show that for every $n \geq 1$,
$$\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2 .$$

2. Use mathematical induction to show that, for every $n \geq 0$,
$4 \cdot 5^n + 7 \cdot (27)^n$ is a multiple of 11.

3. Use mathematical induction to show that, for every $n \geq 0$,
$55 \cdot (44)^n - 6 \cdot (23)^n$ is a multiple of 7 .

4. (Childs, p12, E8) Use mathematical induction to show that for every *odd* number $m \geq 1$,
$4^m + 5^m$ is a multiple of 9 .
(Hint: <u>don't</u> induct on m !)

5. (Childs, p.15, E4) Use complete induction to show that for any convex polygon with n sides, the sum of the angles inside the polygon is (in radians) $(n-2)\pi$. [FYI: convex means, essentially, that the line segment running between any two non-adjacent vertices of the polygon lies entirely on the inside of the polygon.]

6. (Childs, p.18, E3) Suppose that, for some fixed integer N, a set S of integers has the property that $s \leq N$ for every $s \in S$. Show that S has a largest element, i.e., there is an $s_0 \in S$ so that $s \leq s_0$ for every $s \in S$. (Hint: read Childs' hint!)