Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1 (a): Show that \( y = \sin x \) is a solution to the differential equation
\[
y'' + (2 \tan x)y' - y = 0
\]
\[
y = \sin x \\
y' = \cos x \\
y'' = -\sin x
\]
\[
y'' + (2 \tan x)y' - y = -\sin x + (2 \frac{\sin x}{\cos x}) \cos x - \sin x \\
= -\sin x + 2 \sin x - \sin x = 0 \checkmark
\]

1 (b): Use reduction of order to find a second, linearly independent, solution to the differential equation.

\[
y = c(x) \cdot \sin x \\
c(x) = \int \frac{e^{\int 2 \tan x \, dx}}{\cos^2 x} \, dx
\]
\[
\int 2 \tan x \, dx = \ln | \sec x | \\
e^{\int 2 \tan x \, dx} = e^{\ln (\sec x)} \\
= (\sec x)^2 = \cos^2 x
\]
\[
c(x) = \int \frac{\cos^2 x}{\sin^2 x} \, dx = \int \cot^2 x \, dx = \int \csc^2 x - 1 \, dx
\]
\[
= -\cot x - x
\]
\[
\Rightarrow y = c(x) \sin x = (- \frac{\cos x}{\sin x} - x) \sin x = -\cos x - x \sin x
\]
is a second solution.

Check:
\[
y = -\cos x - x \sin x \\
y' = \sin x - \sin x - x \cos x \\
= -x \cos x \\
y'' = -\cos x + x \sin x \\
y'' + (2 \frac{\sin x}{\cos x}) y' - y \\
= -\cos x + x \sin x + (-2x \sin x) + \cos x + x \sin x \\
= (1 - 2x) \sin x = 0 \checkmark
\]