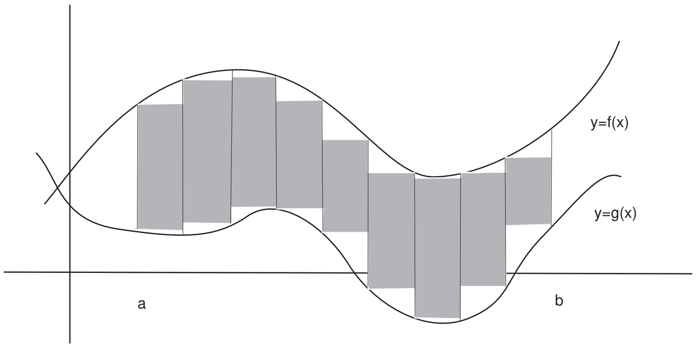
Math 106 Topics Since the Third Exam

Applications of integration:

Area between curves

Region between two curves; approximate by rectangles



Area $= \displaystyle\int_{left}^{right} (top) - (bottom)\ dx = \int_a^b f(x) - g(x)\ dx$

Integrate dy: Area $= \displaystyle\int_{bottom}^{top} (right) - (left)\ dy$

ssk

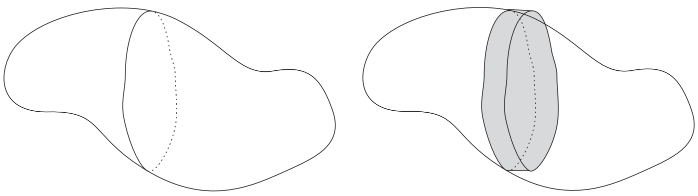
If what the function at top/bottom is changes, cut the interval into pieces, and

use $\displaystyle\int_a^b = \int_a^c + \int_c^b$

Sometimes to calculate area between $f(x)$ and $g(x)$, need to first figure out limits of integration; solve $f(x) = g(x)$, then decide whicxh one is bigger in between each pair of solutions.

Volume by slicing:

To calculate volume, aprroximate region by objects whose volume we can calculate.



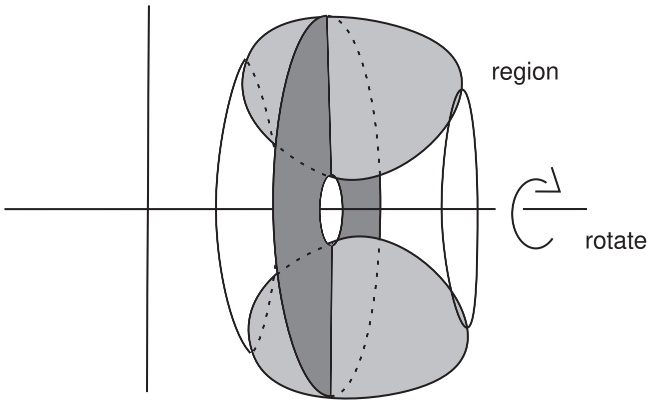
Volume $\approx \sum$(volumes of 'cylinders') $= \sum$(area of base)(height)

$= \sum$(area of cross-section)Δx_i

So volume $= \displaystyle\int_{left}^{right}$ (area of cross section) dx

Solids of revolution: disks and washers:

Solid of revolution: take a region in the plane and revole it in space around an axis in the plane.



take cross-sections perpendicular to axis of revolution

cross-section = disk (area=πr^2) or washer (area=$\pi R^2 - \pi r^2$)

rotate around x-axis: write r (and R) as functions of x, integrate dx

rotate around y-axis: write r (and R) as functions of y, integrate dy

Otherwise, everything is as before:

volume $= \displaystyle\int_{left}^{right} A(x)\ dx$ or volume $= \displaystyle\int_{bottom}^{top} A(y)\ dy$

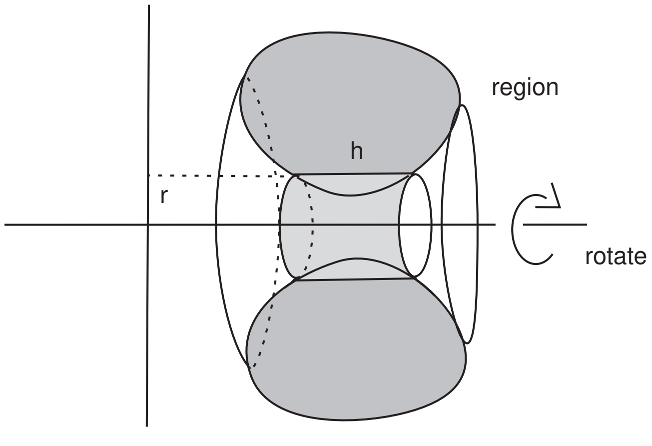
same is true if axis is parallel to $x-$ or $y-$axis; r and R just change (add a constant)

Cylindrical shells:

Different picture, same volume!

Solid of revolution; use cylinders centered on the axis of revolution

Intersection is a cylinder, with area = (circumference)(height) = $2\pi rh$



volume $= \int_{left}^{right}$ (area of cylinder) dx or $\int_{bottom}^{top}$ (area of cylinder) dy!)

revolve around vertical line: integrate dx

revolve around horizontal line: integrate dy

Ex: region in plane between $y = 4x$, $y = x^2$, revolved around y-axis

left=0, right=4, $r = x$, $h = (4x - x^2)$

volume $= \int_0^4 2\pi x(4x - x^2)\ dx$

Lengths of curves:

Idea: approximate a curve by lots of short line segments; length of curve ≈ sum of lengths of line segments.

Line segment between $(c_i, f(c_i))$ and $(c_{i+1}, f(c_{i+1}))$:

$$\sqrt{1 + (\frac{f(c_{i+1}) - f(c_i)}{c_{i+1} - c_i})^2} \cdot (c_{i+1} - c_i) \approx \sqrt{1 + (f'(c_i))^2} \cdot \Delta x_i$$

<u>So</u> length of curve $= \int_{left}^{right} \sqrt{1 + (f'(x))^2}\ dx$

The problem: integrating $\sqrt{1 + (f'(x))^2}$!

sometimes, $1 + (f'(x))^2$ turns out to be a perfect square.....