Math 221, Section 5

Quiz number 5 Solution

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1. Find the solution to the initial value problem

$$2y'' + 5y' + 3y = 0$$

$$y(0) = 3 \qquad , \qquad y'(0) = 4$$

The associated characteristic equation

$$2a^2 + 5a + 3 = 0$$

has roots

$$a = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 2 \cdot 3}}{2 \cdot 2} = \frac{-5 \pm \sqrt{25 - 24}}{4} = \frac{-5 \pm 1}{4}$$

$$\text{so } a = \frac{-5+1}{4} = -1 \text{ or } a = \frac{-5-1}{4} = -\frac{3}{2}.$$

So our fundamental solutions are $y_1 = e^{-x}$ and $y_2 = e^{-\frac{3}{2}x}$, so the solution to our IVP is

$$y = c_1 e^{-x} + c_2 e^{-\frac{3}{2}x},$$

where, since $y' = -c_1 e^{-x} - \frac{3}{2} c_2 e^{-\frac{3}{2}x}$, we need

$3 = y(0) = c_1 e^{-0} + c_2 e^{-\frac{3}{2}0} = c_1 + c_2$ and
$4 = y'(0) = -c_1 e^{-0} - \frac{3}{2} c_2 e^{-\frac{3}{2}0} = -c_1 - \frac{3}{2} c_2$

So we need to find c_1 and c_2 so that $c_1 + c_2 = 3$ and $-c_1 - \frac{3}{2} c_2 = 4$.

Adding the two equations together yields $7 = -\frac{1}{2} c_2$, so $c_2 = -14$.
Then $3 = c_1 + c_2 = c_1 - 14$, so $c_1 = 17$. So

$$y = 17e^{-x} - 14e^{-\frac{3}{2}x}$$

is the solution to our initial value problem.