Math 221, Section 5

Quiz number 3 Solution

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

Find the solution to the initial value problem

$$\frac{dy}{dx} = \frac{2}{x}y + x + 1$$
$$y(1) = 3 \ .$$

Solution: Putting the equation into standard form,

$$y' - \frac{2}{x}y = x + 1,$$

we have (in our standard notation)

$$P(x) = -\frac{2}{x} \text{ and } Q(x) = x + 1 \ .$$

$\int P(x)\ dx = \int -\frac{2}{x}\ dx = -2\ln|x|$, so

$$e^{\int P(x)\ dx} = e^{-2\ln|x|} = e^{\ln|x^{-2}|} = |x^{-2}| = x^{-2}.$$

Then

$$(x^{-2}y)' = x^{-2}(y' - \frac{2}{x}y) = x^{-2}(x+1) = x^{-1} + x^{-2} \ ,$$

so

$x^{-2}y = \int x^{-1} + x^{-2}\ dx = \ln|x| - x^{-1} + c$, so

$$y = x^2(\ln|x| - x^{-1} + c) = x^2\ln|x| - x + cx^2 \ .$$

Setting $x = 1$ and $y = 3$, we have $3 = (1)^2(0) - 1 + c(1)^2 = c - 1$, so $c = 3 + 1 = 4$. So

$$y = x^2\ln|x| - x + 4x^2$$

is the solution to our initial value problem.