Math 208H, Section 1

Practice problems for Exam 1

[Note: These problems were taken from four exams previously given by the instructor. Each of those exams had six (6) problems, which is probably a good indication of the length of your upcoming exam...]

1. Find the **sine** of the angle between the vectors

$$(1,-1,2) \qquad \text{and} \qquad (1,2,1)$$

2. Find a vector of length 3 that is perpendicular to both

$$\vec{v} = \langle 1,3,5 \rangle \text{ and } \vec{w} = \langle 2,1,-1 \rangle \ .$$

3. Show that if the vectors $\vec{\mathbf{v}}=(a_1,a_2,a_3)$ and $\vec{\mathbf{w}}=(b_1,b_2,b_3)$ have the same length, then the vectors

$$\vec{\mathbf{v}} + \vec{\mathbf{w}} \text{ and } \vec{\mathbf{v}} - \vec{\mathbf{w}}$$

are perpendicular to one another.

4. Find the equation of the plane in 3-space which passes through the three points

$$(1,2,1)\ ,\ (6,1,2),\ \text{and}\ (9,-2,1)\ .$$

Does the point $(3,2,1)$ lie on this plane?

5. Find the partial derivatives of the following functions:

(a) $f(x,y,z) = x\tan(2x+yz)$

(b) $g(x,y) = \dfrac{x^2y - ty^4}{\sin(3y)+4}$

6. Find the equation of the tangent plane to the graph of the equation

$$f(x,y,z) = xy^2 + x^2z - xyz = 5$$

at the point $(-1,1,3)$.

7. Calculate the first and second partial derivatives of the function

$$\frac{\sin(x+y)}{y}$$

8. In which direction is the function

$$f(x,y) = x^4y - 3x^2y^2$$

increasing the fastest, at the point (1,2) ? In which direction**s** is the function *neither* increasing *nor* decreasing?

9. If

$$f(x,y) = x^2y^5 - x + 3y - 4\ ,$$

where

$$x = x(u,v) = \frac{u}{u+v} \quad \text{and} \quad y = y(u,v) = uv - u\ ,$$

use the Chain Rule to find $\dfrac{\partial f}{\partial u}$ when $u = 1$ and v=0 .

10. If $f(x, y) = \dfrac{x^2 y}{x + y}$, and $\gamma(t) = (x(t), y(t))$ is a parametrized curve in the domain of f with $\gamma(0) = (2, -1)$ and $\gamma'(0) = (3, 5)$, then what is $\dfrac{d}{dt} f(\gamma(t))\Big|_{t=0}$?

11. Find the **second** partial derivatives of the function

$$h(x, y) = x \sin(xy^2) \ .$$

12. For which value(s) of c are the vectors

$$\vec{v} = (1, 2, c) \text{ and } \vec{w} = (-5, 2c, 4)$$

orthogonal?

13. Find the equation of the plane passing through the points

$$(2, 3, 5) \ , \ (1, -1, 0) \ , \text{ and } (1, 1, 2) \ .$$

14. What is the rate of change of the function

$$f(x, y) = \frac{xy}{x + 2y} \ ,$$

at the point $(4, 2)$, and in the direction of the vector $\vec{v} = (1, 1)$?

15. Find the equation of the plane tangent to the graph of the function

$$g(x, y) = x^3 y - 4x^2 y^2 + 2xy^4$$

at the point $(2, 1, g(2, 1))$.

16. If $x = u^2 v$ and $y = uv^2$, then show how to express the partial derivatives of

$$g(u, v) = f(x(u, v), y(u, v))$$

at the point $(u, v) = (2, -1)$, in terms of the (at the moment unknown) partial derivatives $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$.

17. Find the **second** partial derivatives of the function

$$h(x, y) = xe^{xy}$$

18. Find the local extrema of the function

$$f(x, y) = 2x^4 - 2xy + y^2 \ ,$$

and determine, for each, if it is a local max. local min, or saddle point.

19. Find the critical points of the function

$$f(x, y) = 2xy^2 - x^2 - 8y^2 \ ,$$

and for each, determine if the point is a rel max, rel min, or saddle point.