Math 208H, Section 1

Practice problems for Exam 1

[Note: These problems were taken from four exams previously given by the instructor. Each of those exams had six (6) problems, which is probably a good indication of the length of your upcoming exam...]

1. Find the **sine** of the angle between the vectors

$$(1,-1,2) \qquad \text{and} \qquad (1,2,1)$$

2. Find a vector of length 3 that is perpendicular to both

$$\vec{v} = \langle 1, 3, 5 \rangle \text{ and } \vec{w} = \langle 2, 1, -1 \rangle \ .$$

3. Show that if the vectors $\vec{\mathbf{v}}=(a_1, a_2, a_3)$ and $\vec{\mathbf{w}}=(b_1, b_2, b_3)$ have the same length, then the vectors

$$\vec{\mathbf{v}} + \vec{\mathbf{w}} \text{ and } \vec{\mathbf{v}} - \vec{\mathbf{w}}$$

are perpendicular to one another.

4. Find the equation of the plane in 3-space which passes through the three points

$$(1, 2, 1) \ , \ (6, 1, 2), \text{ and } (9, -2, 1) \ .$$

Does the point $(3, 2, 1)$ lie on this plane?

5. Find the partial derivatives of the following functions:

(a) $f(x, y, z) = x \tan(2x + yz)$

(b) $g(x, y) = \dfrac{x^2y - ty^4}{\sin(3y) + 4}$

6. Find the equation of the tangent plane to the graph of the equation

$$f(x, y, z) = xy^2 + x^2z - xyz = 5$$

at the point $(-1, 1, 3)$.

7. Calculate the first and second partial derivatives of the function

$$\frac{\sin(x + y)}{y}$$

8. In which direction is the function

$$f(x, y) = x^4y - 3x^2y^2$$

increasing the fastest, at the point (1,2) ? In which direction**s** is the function *neither* increasing *nor* decreasing?

9. If

$$f(x,y) = x^2y^5 - x + 3y - 4 \ ,$$

where

$$x = x(u,v) = \frac{u}{u+v} \quad \text{and} \quad y = y(u,v) = uv - u \ ,$$

use the Chain Rule to find $\dfrac{\partial f}{\partial u}$ when $u = 1$ and v=0 .

10. If $f(x,y) = \dfrac{x^2y}{x+y}$, and $\gamma(t) = (x(t), y(t))$ is a parametrized curve in the domain of f with $\gamma(0) = (2,-1)$ and $\gamma'(0) = (3,5)$, then what is $\dfrac{d}{dt} f(\gamma(t))\Big|_{t=0}$?

11. Find the **second** partial derivatives of the function

$$h(x,y) = x\sin(xy^2) \ .$$