Exam 3 Practice problems

3. (15 pts.) Find the arclength of the path
\[ c(t) = \left( \frac{t^2}{2}, \frac{t^3}{3} \right) \]
from \( t = 1 \) to \( t = 3 \)
(Hint: once you have your differential of arc length, factor it.)

3. (20 pts.) Find the curl of the vector field
\[ F(x, y, z) = (2xyz, x^2 - xy^2, 2xyz - yz^2) = (F_1, F_2, F_3) \]
What does this tell us about whether or not \( F \) is a gradient vector field?

7. (20 pts.) Find the line integral of the vector field
\[ F(x, y) = (x^2 - y^2, y^3 - 2xy) \]
along the path \( c(t) = (t, 1 - t) \), \( 0 \leq t \leq 1 \).

9. (20 pts.) Find the area of the region \( D \) in the plane whose boundary is the parametrized curve
\[ c(t) = (4t - t^3, 2t - t^2), \ 0 \leq t \leq 2 \]
(see figure).

1. Find the velocity and acceleration of the parametric curve
\[ (x(t), y(t)) = (t - \sin t, 1 + 2 \cos t) \]

6. Integrate the function \( f(x, y, z) = z \) over the solid region \( R \) bounded by the \( x\)-\( y \) plane (i.e., \( z = 0 \)) and the paraboloid
\[ z = x^2 + y^2 - 9 \]
(see figure). (Hint: a different coordinate system might simplify some of the calculation.)

7. Find the area of the region \( S \) bounded by one loop of the curve described by
\[ r = \sin(3\theta) \]
in polar coordinates; see the figure below. (Hint: to determine the limits of integration, when is \( r = 0 \)?)

8. Find the volume of the region \( T \) lying between the sphere \( \rho = 3 \) (in spherical coordinates) and the cone \( \phi = \pi /6 \); see figure below.