M208H Exam 3 Practice problems

A. (15 pts.) Find the arclength of the path

$$\mathbf{c}(t) = (\frac{t^2}{2}, \frac{t^3}{3})$$

from $t = 1$ to $t = 3$

(Hint: once you have your differential of arc length, **factor it**.).

B. (20 pts.) Find the curl of the vector field

$$F(x, y, z) = (2xyz, x^2 - xy^2, 2xyz - yz^2) = (F_1, F_2, F_3)$$

What does this tell us about whether or not F is a gradient vector field?

C. (20 pts.) Find the line integral of the vector field

$F(x, y) = (x^2 - y^2, y^3 - 2xy)$

along the path $c(t) = (t, 1 - t)$, $0 \leq t \leq 1$.

D. (20 pts.) Find the area of the region D in the plane whose boundary is the parametrized curve

$c(t) = (4t - t^3, 2t - t^2)$, $0 \leq t \leq 2$.

E. Find the velocity and acceleration of the parametric curve

$$(x(t), y(t)) = (t - \sin t, 1 + 2\cos t) \ .$$

F. Find the volume of the region T lying between the sphere $\rho = 3$ (in spherical coordinates) and the cone $\phi = \pi/6$.

G. (20 pts.) Find the integral of the function

$$f(x, y) = x^2yz$$

over the region lying under the graph of the function $z = x^2$ and over the region in the x-y plane with $x^2 + y^2 \leq 4$ and $y \geq 0$.

(Hint: this is probably most easily done $dz\ dy\ dx$).

H. (20 pts.) Find the volume of the region lying under the graph of the function

$$f(x, y) = \cos(x^2 + y^2) + 1$$

which lies over the circle of radius 3 in the x-y plane centered at the origin.

J. (20 pts.) A particle moves along a curve C in 3-space, starting at time $t = 0$ at the point (1,0,1), and at every time t, it's **velocity vector** is given by

$$\vec{r}\,'(t) = (2t, 1, 4t^3)$$

What is the particle's position at time $t = 2$?

(Hint: how do you determine $f(t)$, knowing $f'(t)$ and $f(0)$?)

K. (20 pts.) Show that the vector field

$$\vec{F}(x, y) = (2xy, x^2 - y^2)$$

is a conservative vector field, find a potential function for $\vec{F}$, and use this function to compute the line integral of $\vec{F}$ over the parametrized curve

$$\vec{r}(t) = (t^2 \cos t, t \sin^2 t) \quad , \quad 0 \le t \le \pi$$

L. (20 pts.) Use Green's theorem to compute the line integral of the vector field

$$\vec{F}(x,y) = (2xy, y^2 - x^2)$$

over the curve which follows the line segments from (0,0) to (2,0) to (0,1) to (0,0).

M. (20 pts.) Find the flux integral of the vector field

$$\vec{F}(x,y,z) = (1, x, yz)$$

over that part of the graph of the function

$$z = f(x,y) = xy$$

which lies over the triangle in the plane with vertices (0,0),(1,0), and (1,2) (and using the upward pointing normal for the surface).

N. (25 pts.) Use a change of variables to find the integral of the function

$$f(x,y) = x^2 + x + 3y$$

over the parallelogram P with vertices (0,0), (1,1), (3,1), and (4,2).

(Hint: Find the (linear!) map φ which takes the unit square [0,1]×[0,1] to P.)