M208H Exam 3 Practice problems

A. (15 pts.) Find the arclength of the path
\[ c(t) = \left( \frac{t^2}{2}, \frac{t^3}{3} \right) \]
from \( t = 1 \) to \( t = 3 \)
(Hint: once you have your differential of arc length, factor it.).

B. (20 pts.) Find the curl of the vector field
\[ F(x, y, z) = (2xyz, x^2 - xy^2, 2xyz - yz^2) = (F_1, F_2, F_3) \]
What does this tell us about whether or not \( F \) is a gradient vector field?

C. (20 pts.) Find the line integral of the vector field
\[ F(x, y) = (x^2 - y^2, y^3 - 2xy) \]
along the path \( c(t) = (t, 1-t) \), \( 0 \leq t \leq 1 \).

D. (20 pts.) Find the area of the region \( D \) in the plane whose boundary is the parametrized curve
\[ c(t) = (4t - t^3, 2t - t^2), \ 0 \leq t \leq 2. \]

E. Find the velocity and acceleration of the parametric curve
\[ (x(t), y(t)) = (t - \sin t, 1 + 2 \cos t) \]

F. Find the volume of the region \( T \) lying between the sphere \( \rho = 3 \) (in spherical coordinates) and the cone \( \phi = \pi/6 \).

G. (20 pts.) Find the integral of the function
\[ f(x, y) = x^2yz \]
over the region lying under the graph of the function \( z = x^2 \) and over the region in the \( x-y \) plane with \( x^2 + y^2 \leq 4 \) and \( y \geq 0 \).
(Hint: this is probably most easily done \( dz \ dy \ dx \)).

H. (20 pts.) Find the volume of the region lying under the graph of the function
\[ f(x, y) = \cos(x^2 + y^2) + 1 \]
which lies over the circle of radius 3 in the \( x-y \) plane centered at the origin.

J. (20 pts.) A particle moves along a curve \( C \) in 3-space, starting at time \( t = 0 \) at the point \((1,0,1)\), and at every time \( t \), its velocity vector is given by
\[ \vec{r}'(t) = (2t, 1, 4t^3) \]
What is the particle’s position at time \( t = 2 \)?
(Hint: how do you determine \( f(t) \), knowing \( f'(t) \) and \( f(0) \) ?)

K. (20 pts.) Show that the vector field
\[ \vec{F}(x, y) = (2xy, x^2 - y^2) \]
is a conservative vector field, find a potential function for \( \vec{F} \), and use this function to compute the line integral of \( \vec{F} \) over the parametrized curve
\[ \mathbf{r}(t) = (t^2 \cos t, t \sin^2 t) \quad , \quad 0 \leq t \leq \pi \]

**L. (20 pts.)** Use Green’s theorem to compute the line integral of the vector field
\[ \mathbf{F}(x, y) = (2xy, y^2 - x^2) \]
over the curve which follows the line segments from (0,0) to (2,0) to (0,1) to (0,0).

**M. (20 pts.)** Find the flux integral of the vector field
\[ \mathbf{F}(x, y, z) = (1, x, yz) \]
over that part of the graph of the function
\[ z = f(x, y) = xy \]
which lies over the triangle in the plane with vertices (0,0), (1,0), and (1,2) (and using the upward pointing normal for the surface).

**N. (25 pts.)** Use a change of variables to find the integral of the function
\[ f(x, y) = x^2 + x + 3y \]
over the parallelogram \( P \) with vertices (0,0), (1,1), (3,1), and (4,2).
(Hint: Find the (linear!) map \( \varphi \) which takes the unit square \([0,1] \times [0,1]\) to \( P \).)