Math 208, Section 3

Practice problems for Exam 2

1. Find the integral of the function $f(x, y) = x$ over the region R lying between the graphs of the curves
$$y = x - x^2 \text{ and } y = x - 1 \text{ (see figure).}$$
2. Find the integral of the function $f(x, y, z) = z$ over the region S bounded by the planes
$$x = 0,\ x = y,\ y = 1,\ z = 0 \text{ and}$$
$$\text{the surface } z = x^2 + 1 \text{ (see figure).}$$

4. Find the critical points of the function
$$f(x, y) = x^2 - y^3 + 6xy$$
Describe what the Second Derivative Test says about each critical point.

5. (15 pts.) Calculate the first and second partial derivatives of the function
$$\frac{\sin(x + y)}{y}$$

1. (25 pts.) Find the critical points of the function
$$f(x, y) = x^2 - xy^2 - 4x$$
and determine which of *rel max*, *rel min*, or *saddle point*, each is.

2. (20 pts.) Find the maximum value of the function
$$f(x, y) = x + 2y$$
subject to the constraint $\quad g(x, y) = x^2 + 2y^2 = 5$.

5. Evaluate the following double integrals (10 pts. each):

(a): $\displaystyle\int_0^1 \int_1^2 x^2y - y^2x \, \mathrm{d}x \, \mathrm{d}y$

(b): $\displaystyle\int_0^1 \int_{\sqrt{x}}^1 x\sqrt{y} \, \mathrm{d}y \, \mathrm{d}x$

5. (20 pts.) Evaluate the integral
$$\int_0^1 \int_0^{(1-y^3)^{1/3}} y(1 - x^3)^{1/3} \, \mathrm{d}x\mathrm{d}y$$
by changing the order of integration. (Trust me, you **can't** evaluate it in the order in which it is given!)

2. (20 pts.) Find the local extrema of the function
$$f(x, y) = x^4 - 4xy + y^2 \ ,$$
and determine, for each, if it is a local max. local min, or saddle point.

3. (20 pts.) Find the maximum and minimum values of the function

$$f(x, y) = 2x^2 - y + y^2$$

subject to the constraint

$$g(x, y) = 4x^2 + y^2 \leq 4$$

4. (20 pts.) Evaluate the interated integral

$$\int_0^2 \int_x^2 x^2(y^4 + 1)^{1/3} \, dy \, dx$$

by rewriting the integral to reverse the order of integration.

(Note: the integral *cannot* be evaluated in the order given....)