Math 208H

What you should know and be able to do, for the final

11. Functions of several variables

Find the domain; sketch cross-sections, sketch contour diagrams/level curves.

12. Vectors

Go between $\vec{i}, \vec{j}, \vec{k}$ notation and coordinate notation, Compute length, dot product, cross product. Compute projection of one vector onto another. Find equation of the plane passing through three points; find equation of a plane from a point and normal vector.

13. Differentiation

Compute partial derivatives, gradient, equation for tangent plane to the graph, higher order partial derivatives. Compute the partial derivatives of a composite function using the Chain Rule for several variables. Remember that mixed partials are equal, and gradient vectors are perpendicular to level curves. Find the quadratic approximation to a function of two variables (Taylor polynomial).

14. Optimization

Find the critical points of a function of two or three variables. Use the *Hessian* of f, $f_{xx}f_{yy} - (f_{xy})^2$, to distinguish between local max's, local mins, and saddle points. Find the global max or min over a domain (unconstrained optimization). Find the max or min for one function lying on the level curve of another (constrained optimization - Lagrange multipliers).

15. Integration

Understand that integrals are *sums*. Compute the integral of a function of two variables over a region lying between two curves. Know how to switch from $dy\ dx$ to $dx\ dy$. Compute the triple integral over a region lying under a graph and over a region in the plane. Determine the 'shadow' in the plane of a region in 3-space. Use change of variables and the Jacobian to simplify a multiple integral, by simplifying the region we integrate over. Integrate a function over a cicular region using polar coordinates. Integrate a function over a spherical region using cylindrical and spherical coordinates.

16. Parametrized curves

Sketch a curve from a parametrization; parametrize a circle and a line through a pair of points. Compute the velocity, acceleration, and length of a parametrized curve. Compute, from velocity and initial point, the parametrization of a curve.

17. Vector fields

Understand vector fields as a choice of vector at each point of a domain. Sketch vector fields, e.g., gradient vector fields.

18. Line integrals

Undertand line integral as the sum of dot products of $\vec{F}$ with velocity vectors. Compute using a parametrization of a curve, and using the Fundamental Theorem of Line

Integrals: the integral of a gradient field depends only on the endpoints. Know that gradient fields are path independent. know that path independent vector fields are gradient fields. Compute a potential function by integrating coordinates. Compute the curl of a vector field; use Green's Theorem to compute a line integral over a closed curve (oriented correctly) as the integral of the curl over the region it bounds. Use to compute the area of a region by integrating a field with curl 1 around the boundary.

19. Flux integrals

Understand flux integral as the rate of flow of a fluid through a surface S whose velocity vectors are $\vec{F}$. Understand as integrating $\vec{F} \bullet \vec{n}\ dA$ where $\vec{n}$ is the unit normal to the surface. Know in general that $\vec{n}\ dA$ can be computed using a parametrization $(x(s,t), y(s,t), z(s,t))$, as $(x_s, y_s, z_s) \times (x_t, y_t, z_t)$. Compute flux integrals in the special cases that S is a piece of the graph of a function ($\vec{n}\ dA = (-f_x, -f_y, 1)$, integrate over shadow of S in the plane), a piece of a cylinder (using cylindrical coordinates), or a piece of a sphere (using spherical coordinates).

20. Calculus of vector fields

Compute the divergence of a vector field, understand it as the flux density of the field. Use the Divergence Theorem to compute a flux integral over the boundary of a region as a triple integral over the region.

Compute the curl of a vector field, understand it as the circulation density of the field. Use Stokes' Theorem to compute the line integral of $\vec{F}$ over a closed curve C as the flux integral of curl($\vec{F}$) over a surface with boundary C. Compute the flux integral of a divergence-free vector field (i.e., a curl field) as the flux integral of the field over another surface with the same boundary.