*Math 189H The Joy of Numbers*

## Group Projects

**Overview:** Each student is required to participate in a group project. Each group will choose a different project topic. You can pick your own group members, but a group must have at least two, and no more than four, members. Each project will have both a written and an oral component. These projects can be based on interesting problems or applications that were considered in class, but which were not fully explored, or they can be chosen from the list of suggested topics below. You may also come up with any topic on your own, subject to the instructor's approval, that has a significant number-theoretic component. All participants in a group will receive the same grade, so it is important that each person in the group participate fully and equally. Your project will count 20% toward your final grade.

**Written Component:** Each group will hand in 1 paper. There is no specific page count required; quality is more important than quantity. (A reasonable estimate might be about 5 to 10 pages long, but this need not be strictly adhered to.) Have something to say and say it clearly and concisely. If you are presenting the results of your investigation of some journal article or textbook chapter, you should fill in the missing parts of each argument or proof and do any problems left to the reader. It would be better to go into a small part of some topic in depth and detail, rather than try to cover a large area superficially. This is your opportunity to show that you can read some mathematics on your own and then explain it in writing to your reader. Please do not copy any references - provide appropriate citations.

**Oral Presentations:** You will have 30 minutes to explain to your colleagues about the topic you have researched. Your presentation should be clear and to the point. Choose your examples carefully to illustrate the points you want to make. In a group presentation, all members in the group should have a role and all should be able to answer any questions which arise. You should rehearse your presentation in advance on some fellow students and leave some time for questions and interruptions. Class presentations always take more time than you think they will! Rehearsal will help you to better gauge how much you can accomplish. Depending upon the number of groups formed, we will devote the last 3 or 4 class periods to these presentations.

**Resources:** You may want to browse through some of the books on Number Theory in the math library. The internet has many good sites dealing with number theoretic topics. Using any of the standard search engines should produce good references. (Be careful about using websites as references  some are more reliable than others! If you want to use a website as one of your references, you should clear it with me first.) Im happy to help you find some references if you like.

**Timetable:** Since each topic may be presented by only one group, priority will go to the group who first informs the instructor of their topic choice. You should inform the instructor of the members of your group as soon as possible, and no later than Thursday, October 20. The topic for your group's project should be submitted to the instructor by Thursday October 27. On or shortly after October 27 we will work out a schedule of oral presentations, which will take place starting Tuesday, November 29.

**Some Possible Topics:**

**Mathematics and Magic:** What are some of the many magicians tricks, especially with cards, that are built upon number theoretic facts?

**Mathematics and Juggling:** How do jugglers describe juggling patterns they use, and what new tricks have been introduced into the juggling repertoire as a result of mathematical analysis of juggling?

**The Gregorian Calendar:** How does it work? How long is a complete period before the sequence of days and dates repeats? How can you find the day of the week for any date?

**Mathematics and Music:** What does Number Theory have to do with piano tuning and the "Well-tempered Klavier"? What was Bach's contribution? What number theoretic problems arise in tuning a piano so that it can be played in any key?

**Random Number Generation:** How does the built-in random number generator in your calculator or your favorite computer language or spreadsheet actually work? What are its strengths and shortcomings? How could it be improved?

**Perfect Numbers and Mersenne Primes:** What is the history of perfect numbers? What precisely is the connection between perfect numbers and Mersenne primes? How are Mersenne primes found? What is known about the possibility of odd perfect numbers?

**Factoring Methods:** We have used only trial division to factor numbers. What are some of the other techniques available? The Fermat method and the Monte-Carlo (or Pollard rho) method are both accessible to someone with the background of this course.

**Non-Decimal Number Systems:** How are number systems constructed on bases other than 10? What are the advantages of bases other than 10? Where are other bases in common use?

**Chinese Remainder Theorem:** What does this say, why is it true, and how is it used? How can it be used to perform arithmetic with large numbers?

**Egyptian Fractions:** Our method of writing fractions as ratios of two integers, e.g. $\dfrac{5}{6}$ , is due to Babylonians. The Egyptians used only fractions with numerator 1 and hence would represent $\dfrac{5}{6}$ as a sum of two fractions $\dfrac{1}{2} + \dfrac{1}{3}$ . Can all ordinary fractions be represented as Egyptian fractions? There are many interesting questions here, both arithmetical and historical.

**Continued Fractions:** We haved encountered this distinctive way of representing fractions: $\dfrac{3}{17} = \dfrac{1}{5+\frac{1}{1+\frac{1}{2}}}$ and $\dfrac{24}{31} = \dfrac{1}{1+\frac{1}{3+\frac{1}{2+\frac{1}{3}}}}$ . In fact, every rational number can be written in this form. What about irrational numbers? Why is expressing things in this way useful?

**Geometric Numbers:** We all know the square numbers. What are triangular numbers and pentagonal numbers? What are some of their interesting properties?

**Fibonacci Numbers:** We have introduced these several times; they are a source of many interesting patterns and even have a journal devoted to them. Why are they important? Where do they appear? What are some (more) of their properties?

**Pythagorean Triples:** We've also seen these triple of numbers satisfying $a^2 + b^2 = c^2$. How can they all be generated? What are some of their interesting properties?

**Sums of powers:** What numbers can be written as the sum of two squares? Or of three squares? Or of two cubes? Three cubes? What is known for higher powers? How do you show that some number(s) can't be written in some such way?