EXAMPLE 7

$f(x) = x^2$	Even function: $(-x)^2 = x^2$ for all x; symmetry about y-axis.
$f(x) = x^2 + 1$	Even function: $(-x)^2 + 1 = x^2 + 1$ for all x; symmetry about y-axis (Figure 1.24a).
$f(x) = x$	Odd function: $(-x) = -x$ for all x; symmetry about the origin.
$f(x) = x + 1$	Not odd: $f(-x) = -x + 1$, but $-f(x) = -x - 1$. The two are not equal. Not even: $(-x) + 1 \neq x + 1$ for all $x \neq 0$ (Figure 1.24b).

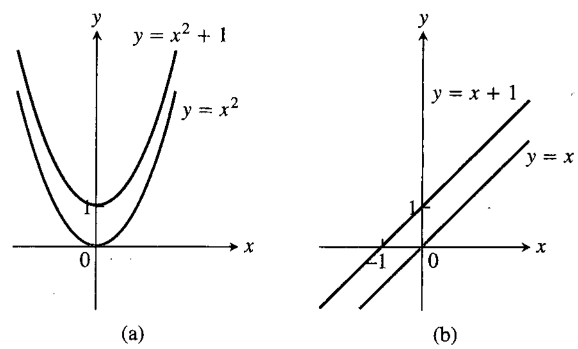


FIGURE 1.24 (a) When we add the constant term 1 to the function $y = x^2$, the resulting function $y = x^2 + 1$ is still even and its graph is still symmetric about the y-axis. (b) When we add the constant term 1 to the function $y = x$, the resulting function $y = x + 1$ is no longer odd. The symmetry about the origin is lost (Example 7) ■

EXERCISES 1.1

In Exercises 1–4, find the domain and range of each function.

1. $f(x) = 1 + x^2$

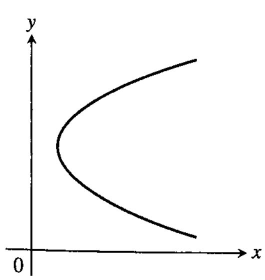
2. $f(x) = 1 - \sqrt{x}$

3. $F(t) = \dfrac{1}{\sqrt{t}}$

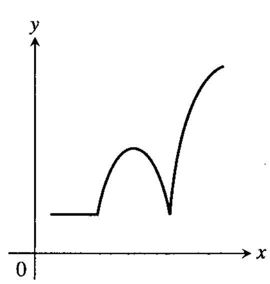
4. $g(z) = \dfrac{1}{\sqrt{4 - z^2}}$

In Exercises 5 and 6, which of the graphs are graphs of functions of x, and which are not? Give reasons for your answers.

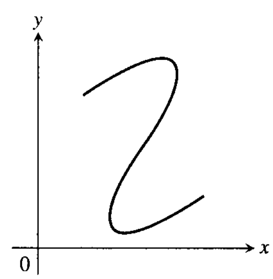
5. a.



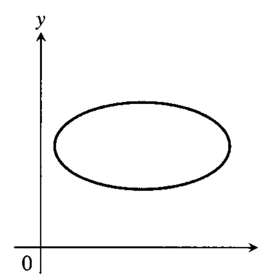
b.



6. a.



b.



7. Express the area and perimeter of an equilateral triangle as a function of the triangle's side length x.

8. Express the side length of a square as a function of the length d of the square's diagonal. Then express the area as a function of the diagonal length.

9. Express the edge length of a cube as a function of the cube's diagonal length d. Then express the surface area and volume of the cube as a function of the diagonal length.

10. A point P in the first quadrant lies on the graph of the function $f(x) = \sqrt{x}$. Express the coordinates of P as functions of the slope of the line joining P to the origin.

Find the domain and graph the functions in Exercises 11–16.

11. $f(x) = 5 - 2x$
12. $f(x) = 1 - 2x - x^2$
13. $g(x) = \sqrt{|x|}$
14. $g(x) = \sqrt{-x}$
15. $F(t) = t/|t|$
16. $G(t) = 1/|t|$

17. Graph the following equations and explain why they are not graphs of functions of x.

   a. $|y| = x$ b. $y^2 = x^2$

18. Graph the following equations and explain why they are not graphs of functions of x.

   a. $|x| + |y| = 1$ b. $|x + y| = 1$

Graph the functions in Exercises 19–22.

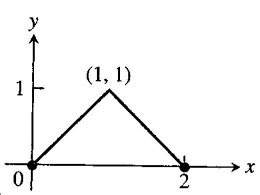
19. $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2 - x, & 1 < x \le 2 \end{cases}$

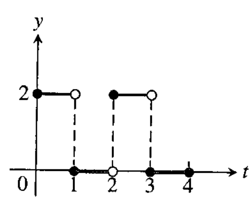
20. $g(x) = \begin{cases} 1 - x, & 0 \le x \le 1 \\ 2 - x, & 1 < x \le 2 \end{cases}$

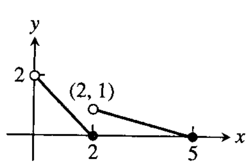
21. $F(x) = \begin{cases} 3 - x, & x \le 1 \\ 2x, & x > 1 \end{cases}$

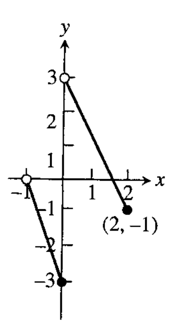
22. $G(x) = \begin{cases} 1/x, & x < 0 \\ x, & 0 \le x \end{cases}$

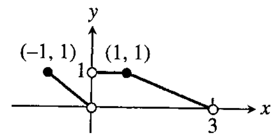
23. Find a formula for each function graphed.

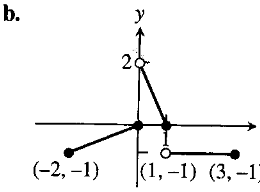
a.


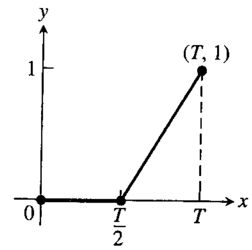
b.


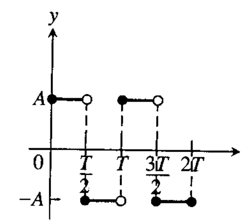
24. a.


b.


25. a.


b.


26. a.


b.


27. For what values of x is

   a. $\lfloor x \rfloor = 0$? b. $\lceil x \rceil = 0$?

28. What real numbers x satisfy the equation $\lfloor x \rfloor = \lceil x \rceil$?

29. Does $\lceil -x \rceil = -\lfloor x \rfloor$ for all real x? Give reasons for your answer.

30. Graph the function

$$f(x) = \begin{cases} \lfloor x \rfloor, & x \ge 0 \\ \lceil x \rceil, & x < 0 \end{cases}$$

Why is $f(x)$ called the *integer part* of x?

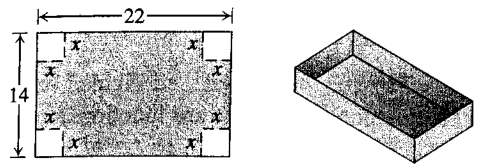
Graph the functions in Exercises 31–42. What symmetries, if any, do the graphs have? Specify the intervals over which the function is increasing and the intervals where it is decreasing.

31. $y = -x^3$
32. $y = -\dfrac{1}{x^2}$
33. $y = -\dfrac{1}{x}$
34. $y = \dfrac{1}{|x|}$
35. $y = \sqrt{|x|}$
36. $y = \sqrt{-x}$
37. $y = x^3/8$
38. $y = -4\sqrt{x}$
39. $y = -x^{3/2}$
40. $y = (-x)^{3/2}$
41. $y = (-x)^{2/3}$
42. $y = -x^{2/3}$

In Exercises 43–54, say whether the function is even, odd, or neither. Give reasons for your answer.

43. $f(x) = 3$
44. $f(x) = x^{-5}$
45. $f(x) = x^2 + 1$
46. $f(x) = x^2 + x$
47. $g(x) = x^3 + x$
48. $g(x) = x^4 + 3x^2 - 1$
49. $g(x) = \dfrac{1}{x^2 - 1}$
50. $g(x) = \dfrac{x}{x^2 - 1}$
51. $h(t) = \dfrac{1}{t - 1}$
52. $h(t) = |t^3|$
53. $h(t) = 2t + 1$
54. $h(t) = 2|t| + 1$

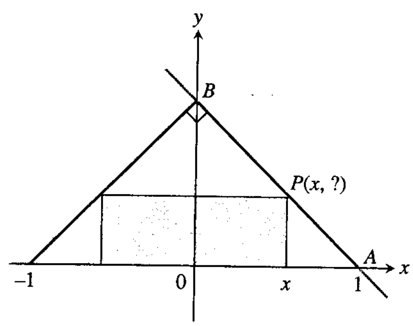
55. A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 14 in. by 22 in. by cutting out equal squares of side x at each corner and then folding up the sides as in the figure. Express the volume V of the box as a function of x.



56. The figure shown here shows a rectangle inscribed in an isosceles right triangle whose hypotenuse is 2 units long.

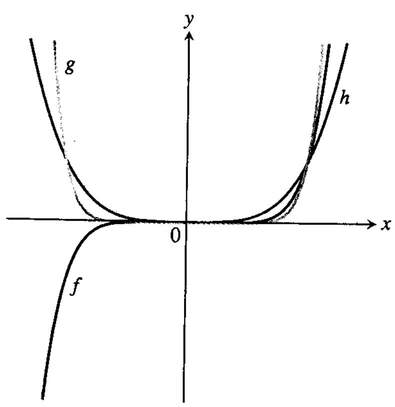
   a. Express the y-coordinate of P in terms of x. (You might start by writing an equation for the line AB.)

   b. Express the area of the rectangle in terms of x.

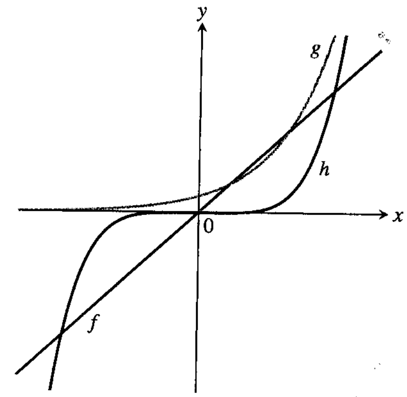


In Exercises 57 and 58, match each equation with its graph. Do not use a graphing device, and give reasons for your answer.

57. a. $y = x^4$ b. $y = x^7$ c. $y = x^{10}$



58. a. $y = 5x$ b. $y = 5^x$ c. $y = x^5$



T 59. a. Graph the functions $f(x) = x/2$ and $g(x) = 1 + (4/x)$ together to identify the values of x for which

$$\frac{x}{2} > 1 + \frac{4}{x}.$$

   b. Confirm your findings in part (a) algebraically.

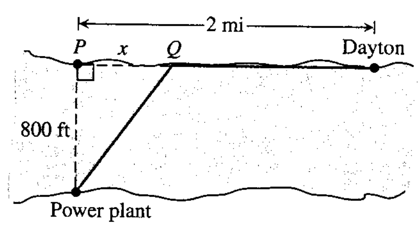
T 60. a. Graph the functions $f(x) = 3/(x - 1)$ and $g(x) = 2/(x + 1)$ together to identify the values of x for which

$$\frac{3}{x - 1} < \frac{2}{x + 1}.$$

   b. Confirm your findings in part (a) algebraically.

61. For a curve to be *symmetric about the x-axis*, the point (x, y) must lie on the curve if and only if the point $(x, -y)$ lies on the curve. Explain why a curve that is symmetric about the x-axis is not the graph of a function, unless the function is $y = 0$.

62. **Industrial costs** Dayton Power and Light, Inc., has a power plant on the Miami River where the river is 800 ft wide. To lay a new cable from the plant to a location in the city 2 mi downstream on the opposite side costs \$180 per foot across the river and \$100 per foot along the land.



(Not to scale)

   a. Suppose that the cable goes from the plant to a point Q on the opposite side that is x ft from the point P directly opposite the plant. Write a function $C(x)$ that gives the cost of laying the cable in terms of the distance x.

   b. Generate a table of values to determine if the least expensive location for point Q is less than 2000 ft or greater than 2000 ft from point P.