RESEARCH STATEMENT

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1. Introduction

Quoting one of its best practitioners, algebra has to do with solving equations. Describing the solutions of linear equations is part of every undergraduate linear algebra course. When dealing with polynomial equations of higher degree, the same task becomes much harder. As an example, consider the problem of characterizing the points on a surface in $\mathbb{R}^4$ that admit a tangent plane (*smooth points*). Algebraic geometry provides tools to overcome our lack of intuition in investigating properties of *algebraic varieties*, i.e. the set of solutions of a system of polynomial equations. Yet problems like finding the minimal number of equations needed to define a curve in a three dimensional space remain unsolved. Following a leitmotif of mathematics, it has been very fruitful to shift the attention from the variety to the functions defined on it. Functions can be added and multiplied, hence they form a *ring*. Geometric properties of varieties translate into algebraic properties of rings, and vice-versa. For example, irreducible algebraic varieties correspond to integral domains. Further, the local properties of a variety are detected by the algebraic properties of rings with a unique maximal ideal, i.e. *local rings*. For example, smooth points correspond to *regular* rings which are part of following hierarchy

$$\text{regular} \subseteq \text{complete intersection} \subseteq \text{Gorenstein} \subseteq \text{Cohen–Macaulay}. \tag{1.0.1}$$

One stream of research in commutative algebra characterizes rings in terms of numerical invariants, which can ultimate be computed with the aid of software. However algebraic geometry is not the only field that influences the development of commutative algebra, as rings arise naturally from number theory, topology, combinatorics, and other fields. In fact, several fundamental theorems were inspired by the work of Hilbert in invariant theory. One of Hilbert's most important contribution to commutative algebra is the syzygy theorem, which foreshadowed the characterization due to Serre of regular local rings. In this context, it became very clear that to study a ring, one has to enlarge the investigation to the class of its *modules*. They are the natural generalization of vector spaces over a field, and techniques from linear algebra find useful applications in module theory.

In the following sections I will describe in more detail my research, which expands into two main directions. The first leads to understand rings where a complete description of a certain module subcategory is available. In the second direction one studies the category of R-modules via an "approximation" with its simplest objects: a *free module* of rank n is the direct sum of n copies of the ring, it is denoted by R^n, and it behaves much like a vector space.

In the following, the rings I consider are local and the unique maximal ideal is denoted by $\mathbf{m}$.

2. Representation type

Over small rings, one can exhaustively describe the module category. For example, over a field the category of finite dimensional vector spaces is parametrized by the integers; over algebras that are finite dimensional vector spaces, is often possible to describe the category of modules by classifying its indecomposable objects. Over larger rings one has to restrict the attention to a suitable subcategory.

2.1. **Cohen-Macaulay Representation Type.** One example of a successful analysis of a subcategory of modules is the one that flourished in the 1980s and led to the study of *maximal Cohen-Macaulay modules* (MCM modules, for short).

A Cohen-Macaulay ring R has *finite Cohen-Macaulay type* if there is a finite number of building blocks in the category of MCM modules, more precisely if the number of isomorphism classes of indecomposable MCM modules is finite. A regular ring has finite Cohen-Macaulay type as the only MCM modules are the free ones. Work of Huneke-Leuschke [15], pioneered by Auslander [2], shows how the finiteness condition on the category of MCM modules reflects heavily on the structure of the ring, and in particular on the character of the singularities:

(2.1) A ring R of finite Cohen-Macaulay type is an *isolated singularity*, i.e. it is regular after localizing at any prime ideal different from the maximal ideal $\mathbf{m}$.

I developed a useful tool in studying this kind of finiteness condition for a given subcategory of R-modules, [26]. Let M and N be two R-modules: Yoneda's definition of the R-module $\operatorname{Ext}^1_R(M, N)$, identifies its elements as short exact sequences

(2.1.1) $$\alpha : 0 \to N \to X_\alpha \to M \to 0,$$

modulo a certain equivalence relation.

Given an R-module X, the set $[X] := \{\alpha \in \operatorname{Ext}^1_R(M, N) \mid X_\alpha \cong X\} \subset \operatorname{Ext}^1_R(M, N)$ does not have a structure of a submodule, but still it gives a lot of information about the R-module $\operatorname{Ext}^1_R(M, N)$. By definition, if X and Y are not isomorphic, then the sets $[X]$ and $[Y]$ are disjoint. In [27], I prove the following

(2.2) **Theorem.** Let R be a local ring and M and N be finitely generated R-modules. If $\operatorname{Ext}^1_R(M, N)$ can be written as a (disjoint) union of finitely many sets $[X]$, then the R-module $\operatorname{Ext}^1_R(M, N)$ has finite length.

This theorem finds application in dealing with a category $\mathcal{C}$ of modules with finitely many isomorphism classes of indecomposable elements: for every M, $N \in \mathcal{C}$ one finds only finitely many modules X that can fit in the middle of a short exact sequence as in 2.1.1, and therefore finitely many sets $[X]$. When $\mathcal{C}$ is the category of MCM modules, a simple proof of 2.1 follows from Theorem 2.2.

Rings of finite Cohen-Macaulay type have been studied extensively: A complete classification of the homogeneous ones is available by work of Eisenbud-Herzog, [10]. In recent work with Crabbe, I show that even over these rings, one cannot hope to impose a finiteness condition on other subcategories of modules.

(2.3) **Theorem.** Let $(R, \mathfrak{m})$ be complete local ring which is not an hypersurface of dimension bigger then 2. For every given integer n, there exists a maximal Cohen-Macaulay module R-module M and a short exact sequence

$$0 \to R/\mathfrak{m}^{s_n} \to X \to M \to 0,$$

such that X is indecomposable, and $X_{\mathbf{p}}$ is free of rank bigger than n for all $\mathbf{p} \neq \mathbf{m}$.

This theorem improves work by Hassler-Wiegand [12], where they construct the module X of arbitrarily large rank over certain hypersurfaces. It is reasonable to think that one could remove the completeness assumption.

2.2. **Simple singularities and totally reflexive modules.** In Theorem 2.1, if we restrict our attention from the class of Cohen-Macaulay rings to the class of Gorenstein rings, a finer description of the kind of singularities is available. Work of Buchweitz, Greuel, Schreyer, Knorrer, Solberg, Herzog and Yoshino explores the relations between Gorenstein rings of finite Cohen-Macaulay type and *simple singularities*, [6,13,34,20,25]. In particular the following holds:

(2.4) Assume R is a complete Gorenstein ring. If R has finite Cohen-Macaulay type, then R is a simple singularity.

The notion of simple singularity is rather technical, but in the context of analytic algebras over a field it comes down to a finite list of families of rings, which were first identified by Arnold [1] in his work on holomorphic germs of function.

One can avoid the condition in 2.4 that the ring R is Gorenstein, by replacing the finiteness condition on the category of MCM modules with a finiteness condition on the category $\mathcal{G}(R)$ of *totally reflexive* modules[1]. Over a Gorenstein ring, the category $\mathcal{G}(R)$ consists exactly of MCM modules; while it is not known that there exists a finitely generated MCM module over a general ring, totally reflexive modules are always available. In [7], in joint work with Christensen, Piepmeyer, and Takahashi, I prove the following

(2.5) **Theorem.** Let R be a complete local ring. If the set of non-isomorphic classes of non-free indecomposable modules in $\mathcal{G}(R)$ is finite and non empty, then R is a simple singularity.

This theorem follows from 2.4 and the following

(2.6) **Theorem.** Let R be a local ring. If the set of indecomposable non-isomorphic modules in $\mathcal{G}(R)$ is finite then either R is Gorenstein or $\mathcal{G}(R)$ consists just of the free R-modules.

The proof finds an application of Theorem 2.2. Theorem 2.6 naturally leads to interesting questions about the category of totally reflexive modules. Paraphrasing it, if a ring is not Gorenstein and there is a non-free totally reflexive module, then there must be infinitely many non-isomorphic indecomposable totally reflexive modules.

(2.7) **Problem.** Let R be a ring which is not Gorenstein and assume there exists a non-free totally reflexive R-module. Find a construction that gives infinitely many non-isomorphic totally reflexive R-modules.

Assume that the length of $\mathrm{Ext}^1_R(G,K)$ is not finite for two given modules G and K which are totally reflexive. Then, by Theorem 2.2, we know that there must be infinitely many non-isomorphic modules X that can fit in the middle of a short exact sequence $0 \to K \to X \to G \to 0$. The module X has to be totally reflexive, and we can construct a family of infinitely many non-isomorphic modules by a push out of the sequence via multiplication with an element of the ring. Unfortunately, this method does not apply, for example, when R is an isolated singularity, and the problem of constructing non-isomorphic totally reflexive R-modules remains open.

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[1] An R-module M is totally reflexive if $\mathrm{Hom}_R(R, \mathrm{Hom}_R(R,M)R)R \cong M$ and $\mathrm{Ext}^i_R(M,R) = 0 = \mathrm{Ext}^i_R(M^*,R)$

It is also clear from Theorems 2.5 and 2.6, that it is interesting to understand when the category of totally reflexive modules consists of free modules, a problem already raised in Avramov-Martsinkovsky [5].

(2.8) **Problem.** Characterize rings in which the category of totally reflexive modules is exactly the category of free R-modules.

Golod rings are a vast class of examples for which any totally reflexive module is free. In [9], Christensen and Veliche show necessary conditions for an artinian ring with $\mathbf{m}^3 = 0$ to guarantee that every totally reflexive module is free. It is not known whether those conditions are also sufficient. Over rings which are *embedded deformations* (quotients of a local ring via a regular sequence) there exists a totally reflexive module which is not free, as shown by Avramov, Gasharov, and Peeva, [4]. The goal of an on-going project with Christensen is to try to determine more necessary conditions for a ring R such that any totally reflexive R module is free in order to try to understand what is the right characterization.

The proof of Theorem 2.6 involves a notion from homological algebra: an R-homomorphism $\phi : X \to M$, is a *$\mathcal{G}(R)$-precover* if $X \in \mathcal{G}(R)$ and every R-homomorphism $\psi : Y \to M$, with $Y \in \mathcal{G}(R)$, can be factored through X. The connection between the notion of precover and Theorem 2.6, is established by the fact that if there are finitely many isomorphism classes of indecomposable R-modules in $\mathcal{G}$, then every R-module M has a $\mathcal{G}$-precover. The following is in [7] and the residue field $\mathsf{k} \cong R/\mathfrak{m}$ plays a special role:

(2.9) **Theorem.** Let $(R, \mathfrak{m}, \mathsf{k})$ be a complete local ring. If k has a $\mathcal{G}$-precover then R is Gorenstein or the category $\mathcal{G}$ consists just of free-modules.

In [32], Takahashi conjectured Theorems 2.9 and 2.6; in [32], [31], and [30] he proved them when the ring R is complete of depth at most two. It is not known whether the ring needs to be complete for the conclusion of Theorem 2.9 to hold.

3. Free resolutions

The idea of a *free resolution* is rather simple. For each R-module M one finds a free module of R^{n_0} that surjects onto M via an R-homomorphism ∂_0; repeating the process of substituting the kernel of ∂_0 for the module M, one obtains a sequence of maps and modules

(3.0.1) $$\cdots \longrightarrow R^{n_i} \xrightarrow{\partial_i} R^{n_{i-1}} \xrightarrow{\partial_{i-1}} \cdots \longrightarrow R^{n_0} \xrightarrow{\partial_0} M,$$

where the image of ∂_i is exactly the kernel of ∂_{i-1} for every $i > 0$. This idea was introduced by Hilbert and in some sense linearizes problems concerning M: the properties of M are intertwined with the properties of the maps ∂_i, which after choosing a basis for the free modules, can be seen as matrices with entries in the ring R. When constructing a free resolution, one can choose a free module of minimal rank; in this case the resolution is said to be *minimal*. The rank of the ith free module R^{n_i} in a minimal resolution is the ith *Betti number* and it is denoted by $\beta_i^R(M)$.

In the late fifties, Auslander, Buchsbaum and Serre [3,24] used resolutions give a positive answer to longstanding conjectures in commutative algebra. In particular, they characterized a local regular ring $(R, \mathfrak{m})$ in terms of the residue field $\mathsf{k} \cong R/\mathfrak{m}$ and its minimal resolution:

(3.1) A local ring R is regular if and only if $\beta_i^R(\mathsf{k}) = 0$ for all i sufficiently large.

Theorem 3.1, shows the important principle that often the residue field works as a test module. Indeed, even when the sequence $\{\beta_i^R(\mathsf{k})\}_i$ does not vanish, it encodes a lot of information about the ring itself. For example,

(3.2) A ring R is a complete intersection if and only if the sequence $\{\beta_i^R(\mathsf{k})\}_i$ has polynomial growth.[2]

The first two classes of rings in the hierarchy 1.0.1 from the introduction are therefore completely characterized by the growth of the Betti numbers. To characterize the third class of rings one studies the sequence of Bass numbers.

3.1. **Bass numbers.** For every integer i, the cohomology modules $\mathrm{Ext}^i_R(\mathsf{k}, R)$ is built from a free resolution of k; its dimension as a vector space over k is called the ith *Bass number* and denoted by μ^i. In analogy with 3.1,

(3.3) A ring is Gorenstein if and only if $\mu^i = 0$ for all i sufficiently large.

On the other hand it is not known if there exists a class of non-Gorenstein Cohen-Macaulay rings for which the sequence of μ^i_R has polynomial growth. In [18], Jorgenson and Leuschke ask whether the sequence $\{\mu^i\}_{i\geq \operatorname{depth} R}$ must have exponential growth for a ring which is not Gorenstein.[3] A positive answer to this question will show an interesting discrepancy between the behavior of the sequence of Bass numbers and the sequence of Betti numbers.

Joint work with Christensen and Veliche, [8], proves the following

(3.4) **Theorem.** Let $(R, \mathfrak{m})$ be a local noetherian ring and set $d = \operatorname{depth} R$. Assume that R is not Gorenstein and one of the following conditions holds:

(1) R is Golod such that either $\mu^d \neq 1$ or the minimal number of the generators of the maximal ideal is bigger then $d + 2$;
(2) R is artinian with $\mathbf{m}^3 = 0$;
(3) R is artinian and a minimal generator of $\mathbf{m}$ is a socle element;
(4) R is the quotient of an artinian Gorenstein ring via its socle element, i.e. R is a Teter ring.

Then the sequence of Bass numbers is strictly increasing and has exponential growth. Further, if (1) holds then the sequence of Bass numbers has term-wise exponential growth.[4]

In [18] the authors show that the sequence $\{\mu^i\}_{i\geq \operatorname{depth} R}$ grows exponentially for some Cohen-Macaulay rings which are not Gorenstein. On the other hand, exponential growth does not imply that the sequence is strictly increasing, and in general this is not the case, as we show in [8]:

(3.5) **Example.** The first four Bass numbers for the ring $R = k[\![x, y]\!]/(x^2, xy)$ are $1, 2, 2, 4$. More in general, $\mu^{\operatorname{depth} R+1} = \mu^{\operatorname{depth} R+2}$, for all Golod non-Gorenstein rings such that $\mathbf{m}$ is minimally generated by $\operatorname{depth} R+2$ elements and $\mu^{\operatorname{depth} R} = 1$. Still, the sequence $\{\mu^i\}_{i\geq \operatorname{depth} R+3}$ has term-wise exponential growth.

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[2] The sequence $\{a_i\}_i$ has polynomial growth if there exists a polynomial $P(n)$ such that $a_i \leq P(i)$.

[3] A sequence $\{a_i\}$ is said to grow exponentially if there is an integer $\alpha > 1$ such that $a_i > \alpha^i$ for all i sufficiently large.

[4] A sequence $\{a_i\}$ is said to grow term-wise exponentially if there exists an integer $\alpha > 1$ such that $a_{i+1} > \alpha a_i$ for all i sufficiently large.

The conclusion of Theorem 3.5 is expected: after all Golod rings can be define as rings for which the Bass numbers are as big as possible. Golod rings which are not Gorenstein could be seen at the antipodes of Gorenstein rings, for which the Bass numbers eventually vanish. In an opposite direction, Teter rings are *almost Gorenstein*, according to a definition introduced by Huneke and Vraciu in [17]. To get a better feeling in understanding the behaviour of the Bass numbers, it would be nice to know whether for all almost Gorenstein rings, which are not Gorenstein, the Bass sequence is strictly increasing and has exponential growth.

Bass numbers have caught the attention of algebraists not only for their growth, but also for some conditions on the values they can attain. In the mentioned work of Jorgenson and Leuschke, particular attention is given to the relation between $\mu^{\operatorname{depth} R}$ and $\mu^{\operatorname{depth} R+1}$; it is asked whether for Cohen-Macaulay rings, $\mu^{\operatorname{depth} R+1} < \mu^{\operatorname{depth} R}$ implies that the ring is Gorenstein. A positive answer to this question is contained in Theorem 3.4 for rings in the list (1)–(4). The relevance of artinian rings with $\mathbf{m}^3 = 0$, lies in the fact that some of the oldest conjectures have been proved true for such rings, see for example the Auslander-Reiten Conjecture [16].

3.2. **Uniform bounds of Artin-Rees type for free resolutions.** When a free resolution involves infinitely many steps, one looks for what possible invariants could be finite. For example, if the ring is graded, one can ask whether the degree of the entries of the matrices representing the maps ∂_i in a free resolution as in 3.0.1 are bounded. In the local setting this leads to the following property

(3.6) Let $(R, \mathbf{m}, \mathsf{k})$ be a local ring and M a finitely generated R-module and $I \subseteq R$ be an ideal. M is *syzygietically Artin-Rees for I* if there exists an integer h such that $I^n R^{\beta_{i-1}} \cap \ker(\partial_i) \subset I^{n-h} \ker(\partial_i)$ for all $n > h$, and for all $i > 0$.

Given an ideal $I \subset R$ and two R-modules $N \subseteq M$, the existance of an integer h such that $I^n M \cap N = I^{n-h}(I^h M \cap N) \subset I^{n-h} N$, for all $n \geq h$, is a classical theorem (the *Artin-Rees Lemma*). Property 3.6 requires an integer h that works simultaneously for the infinitely many modules $\ker \partial_i \subseteq R^{\beta_{i-1}}$. Uniform properties of Artin-Rees type, have been explored in other directions. Given two modules $N \subseteq M$ and a family of ideals $\mathcal{I}$, historically people have been studied whether there exists an integer h that works simultaneously for all ideal $I \in \mathcal{I}$ (see [14], [23], [22], [33]), and this is the direction that first caught my interest [29].

Eisenbud and Huneke in [11] prove that modules which are free when localizing at a prime ideal different from the maximal ideal $\mathbf{m}$ are syzygietically Artin-Rees. Using results from [26], in [28] I prove:

(3.7) **Theorem.** Assume that either R is Cohen-Macaulay or that $\dim R \leq 2$. Let I be an $\mathbf{m}$-primary ideal. Every R-module M is syzygietically Artin-Rees for I.

The uniform Artin-Rees property for free resolution is strictly related to find a uniform annihilator for a certain family of Tor modules.

(3.8) Let M be a finitely generated R-module and let I be an ideal. If M is syzygietically Artin-Rees for I, then there exists an integer h such that $I^h \operatorname{Tor}_j^R(M, R/I^n) = 0$ for every integer n and every $j \geq 0$.

The proof of Theorem 3.7 goes via a converse of 3.8, and it would be interesting to know if such a converse holds in general. In such an investigation, I am looking at a family of rings for which generalizations of the techniques used in the proof of Theorem 3.7 are available.

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