

## Homework 4

This is the list of exercises you can choose from for the third homework assignment. You need to score 30 points. Note I am not grading more than 30 points. Good luck and good fun.

1. (5 points) Find an example of a division ring which is not a field, justify your statement.
2. (10 points) Let  $G$  be a group of order 255. What is the order of a 5-Sylow group? How many 5-Sylow groups can  $G$  have? What is the order of a 3-Sylow group in a group of order 24?
3. (10 points) By using the three Sylow Theorems, show that a group of order 255 must have a normal subgroup.
4. (10 points) By using the three Sylow Theorems, show that a group of order 15 must have a normal subgroup.
5. (15 points) Prove the following
  - (a) Prove that  $\mathbb{Z}_p$  is a field if and only if  $p$  is prime.
  - (b) Prove that  $(\mathbb{Z}_p \setminus \{0\}, +, *)$  is a group, where  $*$  is the usual multiplication. What is the order of such a group?
  - (c) Prove that  $a^{p-1} \equiv 1 \pmod{p}$  for every  $a \in \mathbb{Z}$ .