Math 314: Linear Algebra Spring 2006 Name ______________________

Review: Sections 1.1, 1.2, 1.3, 2.1, 2.2, 2.3, 3.1, 3.2, 3.3, 3.5

1 Vectors, lines and planes

1. Given two vectors in R^n, mark if the following are true or false:
   - If $\mathbf{v} + \mathbf{w} = \mathbf{u} + \mathbf{w}$ then $\mathbf{u} = \mathbf{v}$.
   - If $\mathbf{v} \cdot \mathbf{w} = \mathbf{u} \cdot \mathbf{w}$ then $\mathbf{u} = \mathbf{v}$.
   - If $\mathbf{v}$ is perpendicular to $\mathbf{w}$ and $\mathbf{u}$ is perpendicular to $\mathbf{w}$ then $\mathbf{u}$ is perpendicular to $\mathbf{v}$.
2. Let $\mathbf{v} = [1, 0, 3]$ and $\mathbf{w} = [3, 1, -1]$.
   - Compute $\mathbf{u} = \mathbf{v} + \mathbf{w}$.
   - Apply $\mathbf{u}$ to the point $P(8, 1, 0)$. Compute the final point of $\mathbf{u}$ applied to P.
   - Are $\mathbf{v}$ and $\mathbf{w}$ perpendicular? Justify your answer.
   - Compute $proj_{\mathbf{v}}\mathbf{w}$.
   - Give the normal equation of the plane containing $\mathbf{w}$, $\mathbf{v}$ and the point P.

2 Systems of linear equations, reow reduced echelon form ...

1. Let the following be a system of linear equations

$$x - 3y + 5z = 0 \quad (1)$$
$$2x - 4y + 2z = 0 \quad (2)$$
$$5x - 11y + 9z = 0 \quad (3)$$

   - How many variables?
   - Write the coefficient matrix and the augmented matrix.
   - Without doing any computation can you say if this system is consistent?
   - Find the row reduced echelon form for the cefficient matrix.
   - How many free variables are there and what are those?
   - Solve the system by using the Gauss Jordan elimination.

3 Linear independence

You need to study very well section 2.3. There will be an exercise in which you need to prove some statment about linearly independence. Again, a good review source is the quiz. BE sure you completely understood the quiz.

4 Matrices

In the following exercises A is the matrix

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 2 & 4 & 1 \end{pmatrix}$$

and B is

$$\begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & 0 \\ 0 & 4 & 1 \end{pmatrix}$$

1. Compute AB.
2. Suppose that a matrix C is row equivalent to the following matrix:

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

   Can you decide if C is invertible?
3. A as in number (1)is an invertible matrix. Using the Gauss-Jordan elimination compute the inverse of A.
4. How many solution does have the sistem $A\mathbf{x} = \mathbf{b}$?
5. By using the inverse of A can you give the solution of $A\mathbf{x} = \mathbf{b}$.
6. An example of an elementary matrix.
7. Give an example where CD is not DC, where C and D are square matrices.
8. Is it $(AB)^n = A^n B^n$?. Prove or disprove?
9. Is it $(A^{-1})^n = (A^n)^{-1}$. Prove or disprove.

Extra exercise:

- page 160 numbers 29, 32, 44.
- pade 176 numbers 17,19, 38, 44.

5 Rank

Section 3.5 is very important. Make sure you read it carefully.

1. The vectors $[1, 1, 0, 0]$, $[0, 1, 1, 0]$, $[0, 1, 0, 1]$, $[1, 0, 0, 1]$ are linearly independent. Explain why I can conclude that they are a base of R^4.

2. The following matrix G the 5×7

$$\begin{pmatrix} 2 & 2 & 0 & 2 & -1 & 0 & 4 \\ 0 & 0 & 3 & -3 & 0 & 1 & -6 \\ 1 & 1 & 0 & 1 & 1 & 0 & 5 \\ 0 & 0 & 2 & -2 & 0 & -1 & -4 \\ 1 & 1 & 0 & 1 & -1 & 0 & 1 \end{pmatrix}$$

is row equivalent to the matrix R

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & -1 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

- What is rank(G)? What is rank(R)?
- Find a base for $row(G)$.
- Without computing a base for $col(G)$, give the dimension of $col(G)$.
- Find a base for $col(G)$.
- Find a base for $null(G)$.
- Check that $\dim null(G) + rank(G) = 7$

This last section can be theoretical as much as section 3.1. If there will be proofs in the test or more theoretical questions, they will be out of this section and 3.1. Be sure that you read carefully the book and you understood well the problems of the homework.
Extra exercises:

- page 208 numbers 55, 63, 27.

Good Luck.