

**Review: Eigenvalues and eigenvectors**

1. Give the definition of eigenvector for a matrix  $A$ :
2. Give an example showing why it is useful to know that  $\mathbf{v}$  is an eigenvector.
3. Given an  $n \times n$  matrix  $A$  what is the characteristic polynomial? What is its degree?
4. Given an  $n \times n$  matrix  $A$ , how many different eigenvalues can you find at most? Justify your answer.
5. Let  $\lambda$  be an eigenvalue for the  $n \times n$  matrix  $A$ . Show that the set of all eigenvectors corresponding to  $\lambda$  together with the zero vector form a subspace for  $R^n$ .



11. What does it mean for two  $n \times n$  matrices  $A$  and  $B$  to be similar?

12. Show that two similar  $n \times n$  matrices have the same determinant.

13. Show that the two following matrices are not similar

$$A = \begin{pmatrix} 1 & 3 \\ 1 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix},$$

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix},$$

14. Let  $A$  and  $B$  two  $4 \times 4$  matrices. Assume that  $\text{rank}(A) = 1$  and  $\text{rank}(B) = 2$ . Can  $A$  and  $B$  be similar?

15. Page 307 number 21.

16. Suppose that  $A$  is a  $6 \times 6$  matrix with characteristic polynomial  $c_A(\lambda) = (1 + \lambda)(1 - \lambda)^2(2 - \lambda)^3$ .

(a) Prove that it is impossible to find three linearly independent vectors  $\mathbf{v}_i$ ,  $i = 1, 2, 3$ , such that  $A\mathbf{v}_i = \mathbf{v}_i$ ,  $i = 1, 2, 3$ .

(b) If  $A$  is diagonalizable, what are the dimensions of the eigenspaces  $E_{-1}, E_1, E_2$ .

17. If  $A$  and  $B$  are invertible matrices prove that  $AB$  and  $BA$  are similar.