Math 314: Linear Algebra Spring 2006 Name ________________

Spanning set and linear independence

1. Let $\mathbf{v}, \mathbf{v}_1, \ldots, \mathbf{v}_k$ be $k+1$ vectors in R^n.
   (a) What does it mean for $\mathbf{v}$ to be a linear combination of $\mathbf{v}_1 \ldots, \mathbf{v}_k$?
   (b) What is the spanning set of $\mathbf{v}_1,\ dots, \mathbf{v}_k$?
2. Assume that $\mathbf{v}, \mathbf{w}, \mathbf{u}$ is a base for R^3. Prove that $\mathbf{v}+\mathbf{w}, \mathbf{v}+\mathbf{u}, \mathbf{w}+\mathbf{u}$ is a base for R^3.
3. Prove by contradiction that every subset of a linearly independent set of vectors is linearly independent.
4. Let $\mathbf{v}, \mathbf{v}_1, \ldots, \mathbf{v}_k$ be $k+1$ vectors in R^n. Assume that $\mathbf{v}_1, \ldots, \mathbf{v}_k$ are linearly independent in R^n and $\mathbf{v} = c_1\mathbf{v}_1 + \cdots + c_k\mathbf{v}_k$ with $c-1 \neq 0$. Prove that $\mathbf{v}, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are linearly independent.