

**System of linear equations**

1. What is the rank of a matrix  $A$ ?
2. The equation  $\frac{x^2-y^2}{x-y} = 1$  is not a linear equation. Denote by  $\mathcal{S}(\textit{notlinear})$  the solution set of such equation. Write a linear equation which solution set  $\mathcal{S}(\textit{linear})$  contains  $\mathcal{S}(\textit{not linear})$ . What are the extra solutions in  $\mathcal{S}(\textit{linear})$ .
3. Consider the following augmented matrix  $(A|\mathbf{b})$  which corresponds to a system of three equations in three variables:

$$\left( \begin{array}{cccc} 1 & -1 & -1 & 2 \\ 3 & -3 & 2 & 16 \\ 2 & -1 & 1 & 9 \end{array} \right)$$

- (a) What is the coefficient matrix?
  - (b) Reduce  $(A|\mathbf{b})$  to a row echelon form.
  - (c) Reduce  $(A|\mathbf{b})$  to a reduced echelon form.
  - (d) What is the rank( $A$ )?
  - (e) Find a solution of the system of equations corresponding to  $(A|\mathbf{b})$ .
4. Give an example of a system of linear equations with 4 variable and 2 free variables.