Math 314: Linear Algebra Spring 2006 Name _____

Exam 1

This exam consist of two pages. There are a total of 175 points plus 15 extra credit. Usually, an exercise valued more points requires more time. You are not allowed to use books or notes.
Good Luck.

1. (10 points) Let A be the following matrix:

$$\begin{pmatrix} 0 & 3 & 0 & -3 & 9 \\ 2 & 2 & 0 & 2 & 8 \\ 0 & 2 & 2 & 0 & 10 \end{pmatrix}.$$

   and let R be the following matrix

$$\begin{pmatrix} 1 & 0 & 0 & 2 & 1 \\ 0 & 1 & 0 & -1 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{pmatrix}.$$

   R is the reduced echelon form of A. Find a basis for the subspace $row(A)^{\perp}$.

2. (30 points) Let Q be the followig matrix

$$\begin{pmatrix} 0 & 1/\sqrt{2} & * \\ 1 & 0 & * \\ 0 & 1/\sqrt{2} & * \end{pmatrix}.$$

   Check that the first two columns are orthonormal. Find the third column so that Q is an orthogonal matrix.
   Give Q^{-1}.

3. (30 points) The characteristic polynomial of the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

   is given by $p(\lambda) = (1-\lambda)^2(-\lambda - 2)$. Decide if the matrix is diagonalizable.

4. (30 points) Consider the upper triangolar matrix

$$T = \begin{bmatrix} 9 & 3 & 1 \\ 0 & 4 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

   Justify why T is a diagonalizable matrix. Let D the diagonal matrix which is similar to T. Find D and the invertible matrix P such that $P^{-1}AP = D$.

5. (20 points) Find a basis for $W^{\perp}$, where $W = \{[x, y, z] \mid 2x + 7y - 3z = 0\}$.

6. (15 points) Explain why the determinant of the following matrix is zero:

$$T = \begin{bmatrix} 1 & 3 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 2 & 6 & 2 & 4 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

7. (20 points) Give a subspace W of R^3, a vector $\mathbf{w}$ in W and a vector $\mathbf{v}$ in R^3 such that $\mathbf{v}$ is orthogonal to $\mathbf{w}$ but it is not in $W^{\perp}$. Can this happen in R^2?

8. (20 points) Consider the following two linear transformations.

$$T_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x_3 \\ x_2 + 3x_1 \\ 2x_1 - x_3 \end{bmatrix}, \quad \text{and} \quad T_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_2 - x_1 \\ x_3 - x_2 \end{bmatrix}.$$

   Find the matrices associated to T_1 and T_2 and the matrix associated to T_2T_1.

9. (Extra credit: 15 points) Let A be a 7×9 matrix with $\dim(null(A)) = 4$. Find the dimension of $row(A)$ and $null(A^T)$.