

**Exam 1**

1. The following matrix  $A$  is invertible. Find the inverse.

$$\begin{pmatrix} 3 & 2 & 0 \\ 6 & 6 & 0 \\ 3 & 4 & 1 \end{pmatrix}$$

Let  $\mathbf{b}$  be the vector  $[2, 0, 3]$ .

How many solutions does it have the system  $A\mathbf{x} = \mathbf{b}$ ? List all the solutions.

What is the span of the columns of  $A$ ?

2. Let  $A$  be the following matrix:

$$\begin{pmatrix} 0 & 3 & 0 & -3 & 9 \\ 2 & 2 & 0 & 2 & 8 \\ 0 & 2 & 2 & 0 & 10 \end{pmatrix}.$$

and let  $R$  be the following matrix

$$\begin{pmatrix} 1 & 0 & 0 & 2 & 1 \\ 0 & 1 & 0 & -1 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{pmatrix}.$$

$R$  is the reduced echelon form of  $A$ .

Mark as true or false the following statements:

- $row(A) = row(R)$ .
- $col(A) = col(R)$ .
- $null(A) = null(R)$ .

Find a basis for the vector space  $col(A)$ .

List the free variables for the system  $A\mathbf{x} = \mathbf{b}$  and find a basis for the vector space  $null(A)$ .

Find the  $rank(A)$ .

3. Explain why the rows of a  $3 \times 5$  matrix have to be linearly dependent.
4. Let  $A$  be a matrix which is not the identity and assume that  $A^2 = A$ . By contradiction show that  $A$  is not invertible.
5. Let  $\mathbf{u}, \mathbf{v}, \mathbf{w}$  three linearly independent vectors. Are  $\mathbf{u} - \mathbf{v}, \mathbf{v} - \mathbf{w}, \mathbf{u} - \mathbf{w}$  linearly independent? Explain.
6. Let  $3x + 5y + 7z = 0$  be a plane. Is it the plane perpendicular to the line  $x = 4t + 1, y = -t + 9, z = -t + 190$ ?
7. True or false: Given two matrices  $A$  and  $B$  is it true that  $(A + B)^2 = A^2 + 2AB + B^2$ ?

Good Luck.