

Second Test, Math 314: Linear Algebra.

1. (20 points) Let A an 5×5 matrix. Assume that $A = P^{-1}DP$, where

$$D = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix}$$

and

$$P = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 2 & 2 & 0 & 2 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 2 \end{pmatrix}$$

- (a) Write the eigenvalues of A and the basis for the corresponding eigenspace.

- (b) Write the characteristic polynomial of A .

- (c) Compute $A^{10}\mathbf{v}$ where $\mathbf{v} = [6, 10, 6, 0, 5]$.

- (d) Compute $\det(A)$.

5. (20 points) Compute the projection of the vector $\mathbf{v} = [1, 0, 2]$ over the vector space $W = \text{span} \langle \mathbf{w}, \mathbf{u} \rangle$, where $\mathbf{w} = [1, 1, 0]$ and $\mathbf{u} = [0, 1, 1]$.

6. (15 points) Let A be the following matrix

$$A = \begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix}$$

Find the eigenvalues of A (by hand, do not use a calculator) and for each eigenvalue find a corresponding eigenvector.

7. (13 points) Let \mathcal{F} be the vector space over the real numbers of the continuous real functions in one variable. Prove that $1, e^x, e^{x^2}, e^{x^3}$ are linearly independent.