Math 314: Linear Algebra - Fall 06

Test 1 Name _____

1. (5 points) Let $\mathbf{v}_1, \ldots, \mathbf{v}_k$ be vectors in R^n. Write which condition they have to satisfy to be linearly independent.

2. (5 points) Show that any list of vectors containg the zero vector is linearly dependent.

3. (5 points) List five equivalent conditions for an $n \times n$ matrix A to be invertible.

4. (38 points) Let A and R be the following matrices:

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 3 & 2 \\ 2 & 0 & 4 & 1 & 0 & 5 & 9 \\ 1 & 0 & 2 & 1 & 0 & 4 & 5 \\ 1 & 0 & 2 & 2 & 2 & 7 & 8 \\ 2 & 0 & 4 & 3 & 2 & 11 & 13 \end{pmatrix} \quad R = \begin{pmatrix} 1 & 0 & 2 & 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R is the reduced echelon form of A.

(a) What is the rank of A?

(b) List a basis for row(A).

(c) List a basis for col(A).

(d) List a basis for col(R).

(e) List a basis for null(A).

(f) Mark as True or false:

i. col(A) = col(R).

ii. row(A) = row(R).

iii. null(A) = null(R).

iv. The last column of A is a linear combination of the first four columns of A.

5. (5 points) In a 3×5 matrix, explain why the columns must be linearly dependent.

6. (15 points) Let A be the following matrix

$$A = \begin{pmatrix} 3 & 4 & 1 \\ 0 & 2 & 1 \\ 1 & 0 & 1 \end{pmatrix}.$$

(a) Decide whether A is invertible. If it is invertible, compute the inverse of A.

(b) Let $\mathbf{b}$ be a non-zero vector. Based on your answer for 6a, how many solution does the system $A\mathbf{x} = \mathbf{b}$ have?

7. (5 points) Let A and R be the following matrices:

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 3 & 2 \\ 2 & 0 & 4 & 1 & 0 & 5 & 9 \\ 1 & 0 & 2 & 1 & 0 & 4 & 5 \\ 1 & 0 & 2 & 2 & 2 & 7 & 8 \end{pmatrix} \quad R = \begin{pmatrix} 1 & 0 & 2 & 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

R is the reduced echelon form of A. Is $\text{col}(A) = R^4$? Explain.

8. (12 points) Let $V = \{[x, y] \in R^2 \mid x \geq 0 \quad \text{and} \quad y \geq 0\}$ the set of points which belongs to the first quadrant. Show that for every vectors $\mathbf{u}$ and $\mathbf{v}$ in V then $\mathbf{u} + \mathbf{v}$ belongs to V. Show that V is not a vector subspace of R^2.

9. (10 points) Show that if $A^2 = A$ and A is not the identity matrix then A is not invertible.