1. Let A be a 3×3 matrix, which determinant is 5 and let B be a 3×3, which determinant is 4.
   (a) What is the rank of A?
   (b) What is $\det(2A)$?
   (c) What is $\det(AB)$?
   (d) Can you compute $\det(A+B)$?

2. $\det(A) = 3$, where A be the following matrix

$$\begin{pmatrix} a & b & c & d & e \\ f & g & h & i & l \\ m & n & o & p & q \\ r & s & t & u & v \\ 0 & 1 & 2 & 1 & 0 \end{pmatrix}.$$

   (a) Can the first row be $[0, 1, 2, 1, 0]$?
   (b) Compute the determinant of the matrix

$$\begin{pmatrix} a & b & c & d & e \\ f & g & h & i & l \\ m+3r & n+3s & o+3t & p+3u & q+3v \\ r & s & t & u & v \\ 0 & 1 & 2 & 1 & 0 \end{pmatrix}.$$

3. Find the value of k such that the following matrix is invertible

$$\begin{pmatrix} 0 & k & 0 \\ 1 & 9 & k \\ k & 4 & 3 \end{pmatrix}.$$

4. Let A be a 4×4 matrix such that $A\mathbf{v} = 0$ where $\mathbf{v}$ is the column vector $[1, 2, 4, 1]^T$. What is the determinant of A?

5. Let A be an 4×4 matrix. Let $[1, 2, 1, 0], [1, 0, 1, 0]$ be eigenvectors with eigenvalue 3, and let $[0, 0, 2, 1], [0, 0, 1, 0]$ be eigenvectors with eigenvalue 5.
   (a) Explain why A is diagonalizable.
   (b) $A = P^{-1}DP$, where P is an invertible matrix and D is a diagonal matrix. Find P and D.

6. Compute

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 9 \\ 0 & 0 & 3 \end{pmatrix}^{10}.$$

7. The polynomial $\mathbf{p}(\lambda) = \lambda(\lambda - 3)^2(\lambda - 4)$ is the characteristic polynomial of a square matrix A.
   (a) what is the size of the matrix?

(b) Assume that the eigenspace corresponding to the eigenvalue $\lambda = 4$ has dimension 2. Explain why A is diagonalizable.

8. Let A be similar to B, $(P^{-1}AP = B)$. If $\mathbf{x}$ is an eigenvector for A then $P^{-1}\mathbf{x}$ is an eigenvector for B.

9. Let A be a matrix such that $A^3 = A$. What are the possible eigenvalues for A?

10. Find $W^{\perp}$ where $W = \{(x, y, z) \mid 3x + 4y + 8z = 0, 4x - 1y + z = 0\}$.

11. Let $W = \text{span} < \mathbf{w}_1 = [2, 1, 3, 4], \mathbf{w}_2 = [0, 1, 2, 1] >$ and let $\mathbf{v} = [1, 0, 1, 0]$.

    (a) is $proj_W \mathbf{v} = proj_{\mathbf{w}_1}\mathbf{v} + proj_{\mathbf{w}_2}\mathbf{v}$?

    (b) Compute an orthogonal basis for W with the Gram-Schmidt algorithm.

    (c) Compute $proj_W \mathbf{v}$.

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13. Say which of the following is a vector space over the real numbers:

    (a) S is the set of real valued functions continuos over R such that $\lim_{x\to 0} f(x) = 0$.

    (b) S is the set of real valued functions continuous over R such that $\int_1^3 f(x)dx$ does exist.

    (c) S is the set of polynomials $p(x)$ such that $p(2) = 0$.

14. Let $\mathcal{F}$ be the vector space of real continuos functions. Are $1, \cos(x), \sin(x) \in \mathcal{F}$ linearly independent?

15. Let $\mathcal{F}$ be the vector space of real continuos functions. Are $1, \cos^2(x), \sin^2(x) \in \mathcal{F}$ linearly independent?