Math 314: Review

1. Prove that any set of vectors containing the zero vector is linearly dependent.
2. Give the definition of a basis of a vector space.
3. Let V be a subspace spanned by vectors in R^n. Prove that $\dim(V) \leq n$.
4. Find a basis for the vector space spanned by the following vector of R^4:
$$[1,2,3,1],[1,0,0,1],[1,2,1,1],[1,0,0,1],[3,1,4,1].$$
5. Check whether the vector $[1,2,4]$ belongs to the vector space spanned by $[1,0,1],[0,1,1]$.
6. Let A be a $m \times n$ matrix and let R the following matrix:
$$\begin{pmatrix} 1 & 0 & 1 & 3 & 1 \\ 0 & 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$
Assume that R is the reduced echelon form of A.
   (a) What is the rank of A?
   (b) Find a basis for row(A).
   (c) What is the dimension of col(A)?
   (d) Can you find a basis for col(A)?
   (e) Can you say which columns of A form a basis for col(A)?
   (f) Find a basis for the null space of A.
7. Let A be an 5×5 matrix, such that the reduced echelon form has 2 zero rows. Explain why the colums must be linearly dependent.
8. Decide whether $S = \{[x,y,z] \mid x^2 = 1\}$ is a vector subspace of R^3.
9. Let A be an $n \times n$ matrix such that $A^2 = A$ and $A \neq I$, where I is the identity matrix. Prove that the rank of A is strictly less than n.
10. Prove that the null space of an $m \times n$ matrix A is a vector subspace of R^n.
11. Let A be an $n \times n$ matrix such that $A^2 = 0$, argue that col$(A) \subseteq$ null(A).
12. Let A be the following matrix
$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$
Decide whether A is invertible, by using the fundamental theorem of linear algebra.
13. Compute the inverse of the matrix A given in 12.

14. Give an example of two matrices A and B such that $(A+B)^2 \neq A^2 + 2AB + B^2$.

15. number 43 page 208.

16. (Extra Credit) Let A and B two $n \times n$ matrices. Argue that $\text{rank}(AB) \leq \text{rank}(A)$, $\text{rank}(AB) \leq \text{rank}(B)$, $\text{rank}(A+B) \leq \text{rank}(A) + \text{rank}(B)$.