Math 314: Linear Algebra - Fall 06
On Eigenvalues and eigenvectors

Name ______________________________

1. Assume that $\lambda = 3$ is an eigenvalue for the matrix B. Explain why you can tell that there exists a nonzero vector $\mathbf{v}$ such that $A\mathbf{v} = 3\mathbf{v}$. (In other words explain why you always know that the geometric multiplicity is always bigeer then one).

2. Let A be the following matrix.

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

   - Write the characteristic polynomial of A.
   - Compute the eigenvalues of the matrix A and for each of them say which is the algebraic multiplicity.
   - For each eigencalue compute a basis for the eigenspace and say what is the geometric multiplicity.
   - Write $[5, 0, 1]$ as linear combination of the vectors you find in .
   - Without using a calculator compute $A^{100}\mathbf{v}$ where $\mathbf{v} = [5, 0, 1]$.

3. Let A be a matrix such that $A^2 = A$, show that the only possible eigenvalues are $\lambda = 0$ and $\lambda = 1$.